

# THE RANK 2 ROOTS PACKAGE

## VERSION 1.1

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### 1. INTRODUCTION

This package concerns mathematical drawings arising in representation theory. The purpose of this package is to ease drawing of rank 2 root systems, with Weyl chambers, weight lattices, and parabolic subgroups, mostly imitating the drawings of Fulton and Harris [2]. We use definitions of root systems and weight lattices as in Carter [1] p. 540–609.

Load the `rank-2-roots` package

```
\documentclass{amsart}
\usepackage{rank-2-roots}
\begin{document}
The root system  $(G_2)$ :
\begin{tikzpicture}[baseline=-.5]
\begin{rootSystem}{G}
\roots
\end{rootSystem}
\end{tikzpicture}
\end{document}
```

### 2. ROOT SYSTEMS



Table 1: The root systems

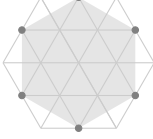

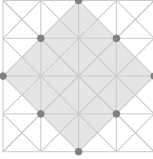
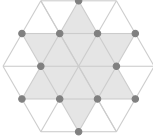
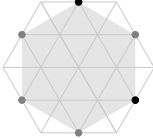

$A_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \roots \end {rootSystem} \end {tikzpicture} </pre>
$B_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \roots \end {rootSystem} \end {tikzpicture} </pre>
$C_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \roots \end {rootSystem} \end {tikzpicture} </pre>
$G_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \roots \end {rootSystem} \end {tikzpicture} </pre>

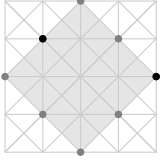
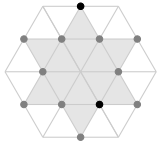
Table 2: The root systems with the simple roots marked

$A_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \simpleroots \end {rootSystem} \end {tikzpicture} </pre>
$B_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \simpleroots \end {rootSystem} \end {tikzpicture} </pre>

continued ...



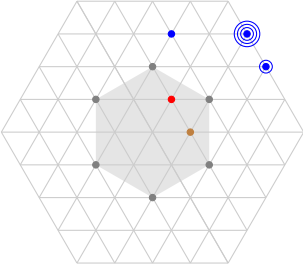
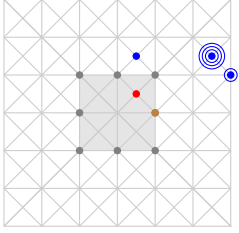
Table 2: ...continued

$C_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \simpleroots \end {rootSystem} \end {tikzpicture} </pre>
$G_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \simpleroots \end {rootSystem} \end {tikzpicture} </pre>

## 3. WEIGHTS

Type `\wt{x}{y}` to get a weight at position  $(x,y)$  (as measured in a basis of *fundamental weights*). Type `\wt[multiplicity=n]{x}{y}` to get multiplicity  $m$ . Add an option: `\wt[Z]{x}{y}` to get Z passed to TikZ.

Table 3: Some weights drawn with multiplicities

$A_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [multiplicity=4,blue]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>
$B_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [multiplicity=4,blue]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>

continued ...



Table 3: ...continued

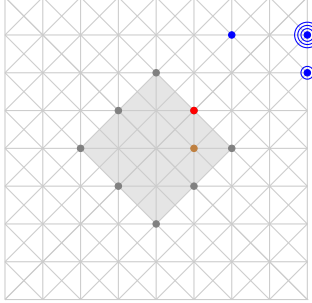
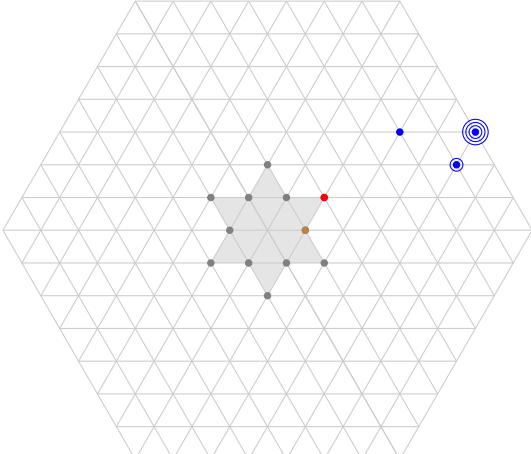
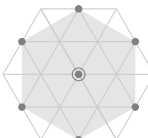

$C_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [multiplicity=4,blue]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>
$G_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [multiplicity=4,blue]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>

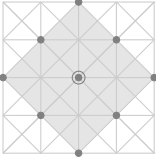
Table 4: The root systems with all multiplicities of the adjoint representation, like Fulton and Harris

$A_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \wt [multiplicity=2,root]{0}{0} \end {rootSystem} \end {tikzpicture} </pre>
$B_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \wt [multiplicity=2,root]{0}{0} \end {rootSystem} \end {tikzpicture} </pre>

continued ...



Table 4: ...continued

$C_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \wt [multiplicity=2,root]{0}{0} \end {rootSystem} \end {tikzpicture} </pre>
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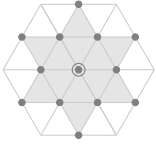
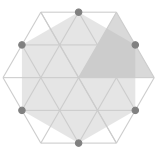

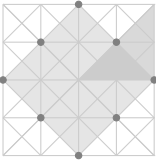
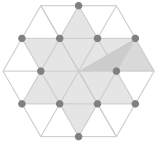
$G_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \wt [multiplicity=2,root]{0}{0} \end {rootSystem} \end {tikzpicture} </pre>
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Table 5: Weyl chambers

$A_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>
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$B_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>
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$C_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>
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$G_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>
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## 4. PARABOLIC SUBGROUPS

Table 6: The positive root hyperplane

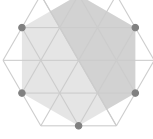

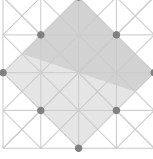
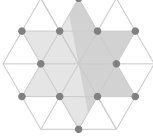
$A_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>
$B_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>
$C_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>
$G_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>

Table 7: Parabolic subgroups. Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:  $A_{5,37}$  means the parabolic subgroup of  $A_5$  so that the binary digits of  $37 = 2^5 + 2^2 + 2^0$  give us roots 0, 2, 5 in Bourbaki ordering being compact roots, i.e. having the root vectors of both that root and its negative inside the parabolic subgroup.

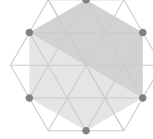
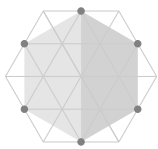
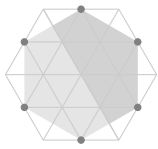



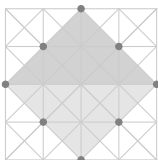
$A_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre> <p>continued ...</p>
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Table 7: ...continued

$A_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>
$A_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>
$C_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre>

continued ...



Table 7: ...continued

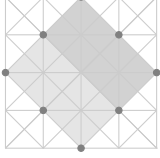
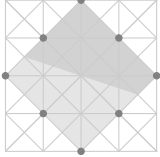
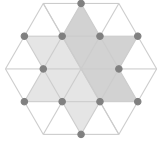
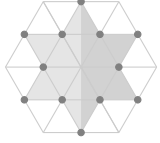
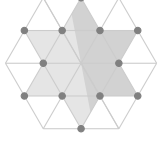
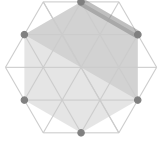
$C_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>
$C_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>

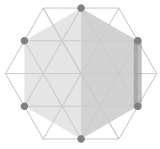
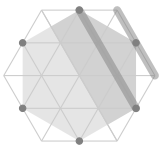
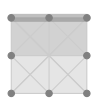
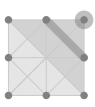
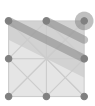
Table 8: Parabolic subgroups with grading of the positive roots

$A_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {1} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
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continued ...



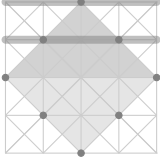
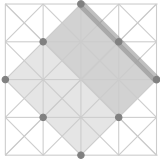
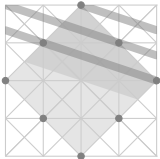
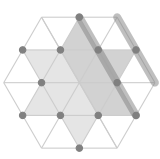
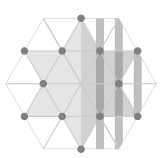
Table 8: ...continued

$A_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {2} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$A_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {3} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {1} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {2} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {3} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>

continued ...



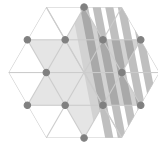
Table 8: ...continued

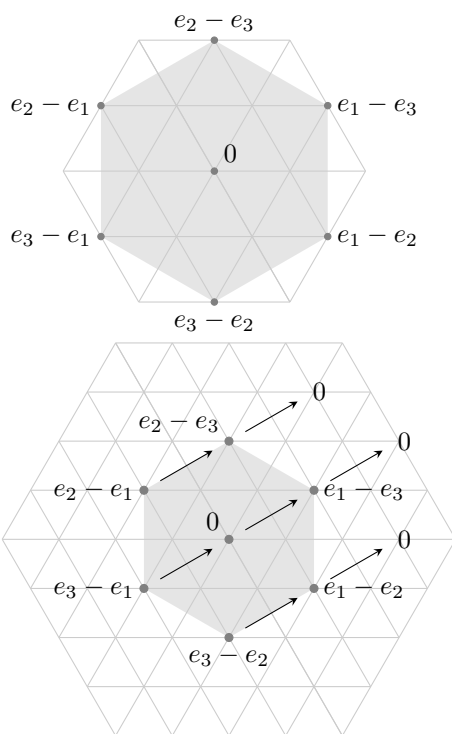
$C_{2,1}$		<pre> \begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{C} \roots \parabolic {1} \parabolicgrading \end{rootSystem} \end{tikzpicture} </pre>
$C_{2,2}$		<pre> \begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{C} \roots \parabolic {2} \parabolicgrading \end{rootSystem} \end{tikzpicture} </pre>
$C_{2,3}$		<pre> \begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{C} \roots \parabolic {3} \parabolicgrading \end{rootSystem} \end{tikzpicture} </pre>
$G_{2,1}$		<pre> \begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{G} \roots \parabolic {1} \parabolicgrading \end{rootSystem} \end{tikzpicture} </pre>
$G_{2,2}$		<pre> \begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{G} \roots \parabolic {2} \parabolicgrading \end{rootSystem} \end{tikzpicture} </pre>

continued ...



Table 8: ...continued

$G_{2,3}$		<pre>\begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{G} \roots \parabolic{3} \parabolicgrading \end{rootSystem} \end{tikzpicture}</pre>
-----------	---	--



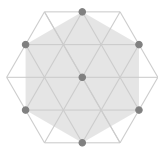
Drawing the  $A_2$  root system and a weight at the origin. The option `root` indicates that this weight is to be coloured like a root.

```

\begin{tikzpicture}
\begin{rootSystem}{A}
\roots
\wt[root]{0}{0}
\end{rootSystem}
\end{tikzpicture}

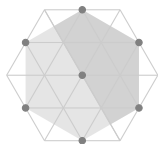
```





Drawing the  $A_2$  root system and a weight at the origin and the positive root hyperplane

```
\begin{tikzpicture}
\begin{rootSystem}{A}
\roots
\wt[root]{0}{0}
\positiveRootHyperplane
\end{rootSystem}
\end{tikzpicture}
```



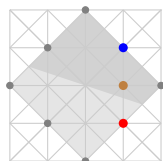
## 5. COORDINATE SYSTEMS

The package provides three coordinate systems: hex, square and weight. Above we have seen the weight coordinates: a basis of fundamental weights. We can also use weight coordinates like

```
\draw \weight{0}{1} -- \weight{1}{0};
```

Drawing weights as linear combinations of fundamental weights

```
\begin{tikzpicture}
\begin{rootSystem}{C}
\roots
\positiveRootHyperplane
\fill[thick,brown] \weight{1}{0} circle (1.7pt);
\fill[thick,blue] \weight{0}{1} circle (1.7pt);
\fill[thick,red] \weight{2}{-1} circle (1.7pt);
\end{rootSystem}
\end{tikzpicture}
```

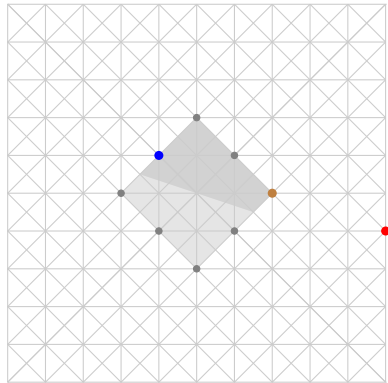




We can also specify roots in linear combinations of the simple roots:

Drawing roots as linear combinations of simple roots

```
\begin{tikzpicture}
\begin{rootSystem}{C}
\roots
\positiveRootHyperplane
\fill[thick,brown] \Root{1}{0} circle (1.7pt);
\fill[thick,blue] \Root{0}{1} circle (1.7pt);
\fill[thick,red] \Root{2}{-1} circle (1.7pt);
\end{rootSystem}
\end{tikzpicture}
```

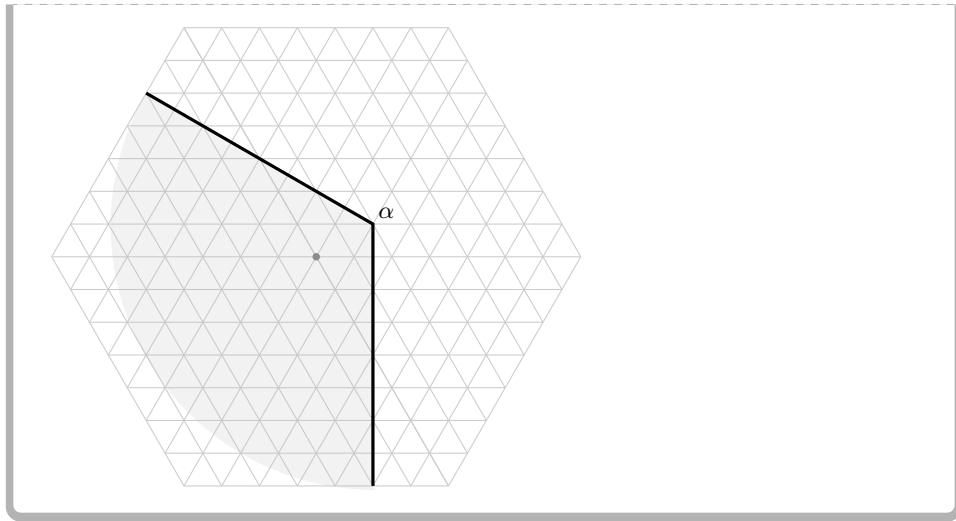


The square system, used like `\draw (square cs:x=1,y=2) circle (2pt);`, is simply the standard Cartesian coordinate system measured so that the minimum distance between weights is one unit. The hex coordinate system has basis precisely the fundamental weights of the  $A_2$  lattice. We can use the hex system in drawing on the  $A_2$  or  $G_2$  weight lattices, as below, as they are the same lattices.

Automatic sizing of the weight lattice (the default) ...

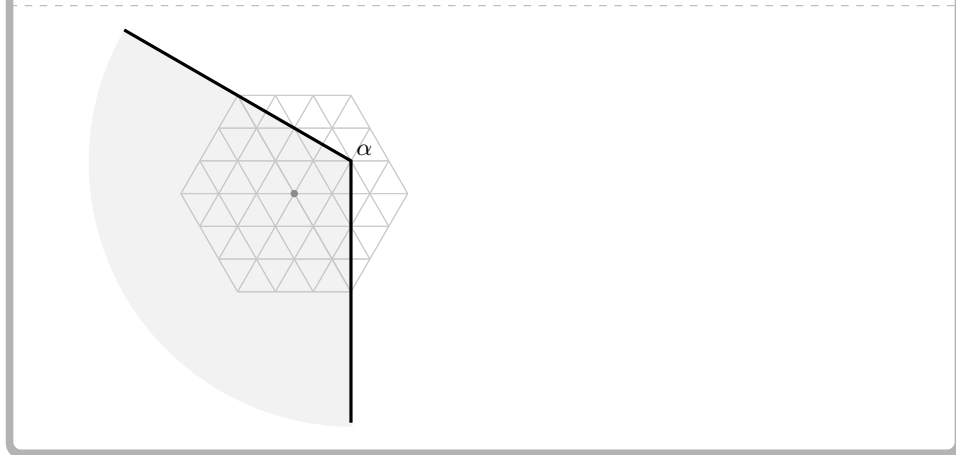
```
\begin{tikzpicture}
\begin{rootSystem}{A}
\wt{0}{0}
\fill[gray!50,opacity=.2] (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
(hex cs:x=-7,y=5) arc (150:270:{7*\weightLength});
\draw[black,very thick] (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
(hex cs:x=-7,y=5);
\node[above right=-2pt] at (hex cs:x=1,y=1) {\small\(\alpha\)};
\end{rootSystem}
\end{tikzpicture}
```





...and here with manual sizing, setting the weight lattice to include 3 steps to the right of the origin

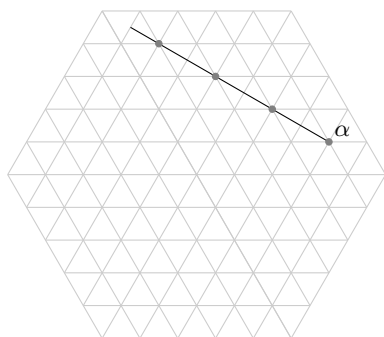
```
\begin{tikzpicture}
\AutoSizeWeightLatticefalse
\begin{rootSystem}{A}
\wt{0}{0}
\weightLattice{3}
\fill[gray!50,opacity=.2] (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
(hex cs:x=-7,y=5) arc (150:270:{7*\weightLength});
\draw[black,very thick] (hex cs:x=5,y=-7) -- (hex cs:x=1,y=1) --
(hex cs:x=-7,y=5);
\node[above right=-2pt] at (hex cs:x=1,y=1) {\small\(\alpha\)};
\end{rootSystem}
\end{tikzpicture}
```





Fulton and Harris p. 170

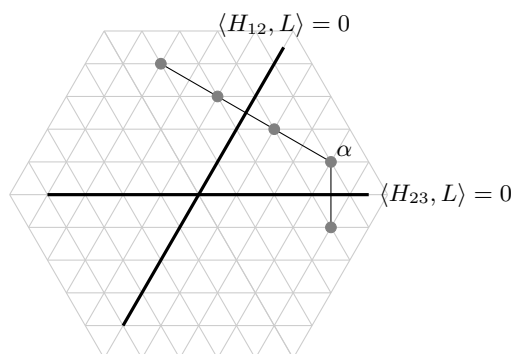
```
\begin{tikzpicture}
\begin{rootSystem}{A}
\draw \weight{3}{1} -- \weight{-4}{4.5};
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
\end{rootSystem}
\end{tikzpicture}
```



Automatic sizing of the weight lattice (the default) ...

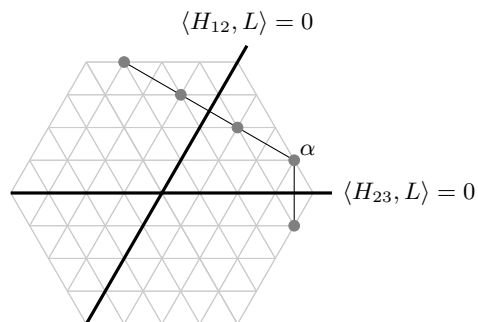
```
\begin{tikzpicture}
\begin{rootSystem}{A}
\setlength{\weightRadius}{2pt}
\draw \weight{3}{1} -- \weight{-3}{4};
\draw \weight{3}{1} -- \weight{4}{-1};
\wt{4}{-1}
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
\draw[very thick] \weight{0}{-4} -- \weight{0}{4.5}
node[above]{\small\(\left<H_{12},L\right>=0\)};
\draw[very thick] \weight{-4}{0} -- \weight{4.5}{0}
node[right]{\small\(\left<H_{23},L\right>=0\)};
\end{rootSystem}
\end{tikzpicture}
```





...and manual sizing

```
\begin{tikzpicture}
\AutoSizeWeightLatticefalse
\begin{rootSystem}{A}
\setlength{\weightRadius}{2pt}
\weightLattice{4}
\draw \weight{3}{1} -- \weight{-3}{4};
\draw \weight{3}{1} -- \weight{4}{-1};
\wt{4}{-1}
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
\draw[very thick] \weight{0}{-4} -- \weight{0}{4.5}
node[above]{\small\(\langle H_{12}, L \rangle = 0\)};
\draw[very thick] \weight{-4}{0} -- \weight{4.5}{0}
node[right]{\small\(\langle H_{23}, L \rangle = 0\)};
\end{rootSystem}
\end{tikzpicture}
```



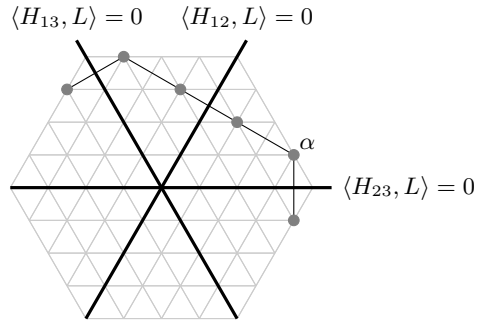
```
\begin{tikzpicture}
\AutoSizeWeightLatticefalse
\begin{rootSystem}{A}
```



```

\setlength{\weightRadius}{2pt}
\weightLattice{4}
\draw \weight{3}{1} -- \weight{-3}{4};
\draw \weight{3}{1} -- \weight{4}{-1};
\draw \weight{-3}{4} -- \weight{-4}{3};
\wt{4}{-1}
\wt{-4}{3}
\foreach \i in {1,...,4}{\wt{5-2*\i}{\i}}
\node[above right=-2pt] at (hex cs:x=3,y=1){\small\(\alpha\)};
\draw[very thick] \weight{0}{-4} -- \weight{0}{4.5}
node[above]{\small\(\left<H_{12},L\right>=0\)};
\draw[very thick] \weight{-4}{0} -- \weight{4.5}{0}
node[right]{\small\(\left<H_{23},L\right>=0\)};
\draw[very thick] \weight{4}{-4} -- \weight{-4.5}{4.5}
node[above]{\small\(\left<H_{13},L\right>=0\)};
\end{rootSystem}
\end{tikzpicture}

```

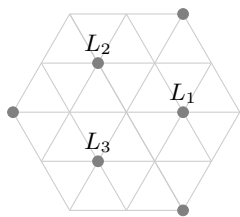


```

\setlength{\weightRadius}{2pt}
\setlength{\weightLength}{.75cm}
\begin{tikzpicture}
\begin{rootSystem}{A}
\foreach \x/\y in {1/0, -1/1, 0/-1, -2/0, 0/2, 2/-2}{\wt{\x}{\y}}
\node[above] at \weight{1}{0} {\small\(\alpha_1\)};
\node[above] at \weight{-1}{1} {\small\(\alpha_2\)};
\node[above] at \weight{0}{-1} {\small\(\alpha_3\)};
\end{rootSystem}
\end{tikzpicture}

```



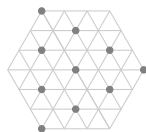


Changing the weight length rescales

```
\begin{tikzpicture}
\setlength\weightLength{.3cm}
\begin{rootSystem}{A}
\wt[multiplicity=2]{0}{0}
\foreach \x/\y in {1/1, 2/-1, 1/-2, -1/-1, -2/1, -1/2}{\wt{\x}{\y}}
\end{rootSystem}
\end{tikzpicture}
```

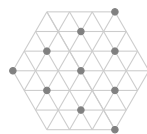


```
\begin{tikzpicture}
\setlength\weightLength{.3cm}
\begin{rootSystem}{A}
\foreach \x/\y in {0/0, 3/0, 2/-1, 1/-2, 0/-3, 1/1, -1/-1, -1/2,
-2/1, -3/3}{\wt{\x}{\y}}
\end{rootSystem}
\end{tikzpicture}
```



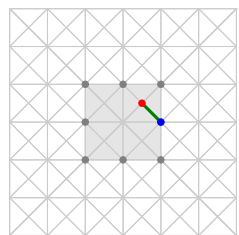
```
\begin{tikzpicture}
\setlength\weightLength{.3cm}
\begin{rootSystem}{A}
\foreach \x/\y in {0/0, -3/0, 2/-1, 1/-2, 3/-3, 1/1, -1/-1, -1/2,
-2/1, 0/3}{\wt{\x}{\y}}
\end{rootSystem}
\end{tikzpicture}
```





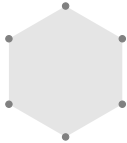

We use a basis of fundamental weights, as given in Carter's book [1] p. 540–609

```
\begin{tikzpicture}
\begin{rootSystem}{B}
\roots
\draw[green!50!black,very thick] \weight{0}{1} -- \weight{1}{0};
\weightLattice{3}
\wt[blue]{1}{0}
\wt[red]{0}{1}
\end{rootSystem}
\end{tikzpicture}
```



Without automatic stretching of the weight lattice to fit the picture, you won't see the weight lattice at all unless you ask for it.

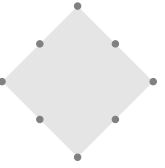

Table 9: The root systems

$A_2$	 <pre>\begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{A} \roots \end{rootSystem} \end{tikzpicture}</pre>
$B_2$	 <pre>\begin{tikzpicture}[baseline=-.5] \begin{rootSystem}{B} \roots \end{rootSystem} \end{tikzpicture}</pre>

continued ...

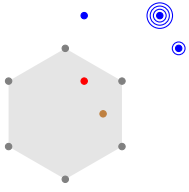
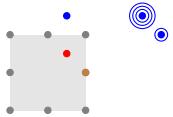


Table 9: ...continued

$C_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \end {rootSystem} \end {tikzpicture} </pre>
$G_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \end {rootSystem} \end {tikzpicture} </pre>

Type `\wt{x}{y}` to get a weight at position  $(x,y)$  (as measured in a basis of *fundamental weights*). Add an option: `\wt[Z]{x}{y}` to get  $Z$  passed to TikZ, or with option `multiplicity=n` to get multiplicity  $n$ .

Table 10: Some weights drawn with multiplicities

$A_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [blue,multiplicity=4]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>
$B_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [blue,multiplicity=4]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>

continued ...



Table 10: ...continued

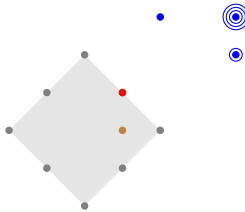
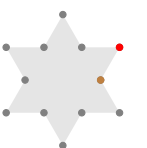
$C_2$ 	<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [blue,multiplicity=4]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>
$G_2$ 	<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \wt [brown]{1}{0} \wt [red]{0}{1} \wt [blue,multiplicity=4]{1}{3} \wt [blue,multiplicity=2]{2}{2} \wt [blue]{-1}{3} \end {rootSystem} \end {tikzpicture} </pre>

Table 11: The root systems with all multiplicities of the adjoint representation, like Fulton and Harris

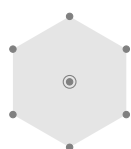
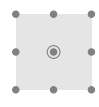
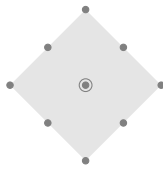
$A_2$ 	<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \wt [multiplicity=2]{0}{0} \end {rootSystem} \end {tikzpicture} </pre>
$B_2$ 	<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \wt [multiplicity=2]{0}{0} \end {rootSystem} \end {tikzpicture} </pre>
$C_2$ 	<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \wt [multiplicity=2]{0}{0} \end {rootSystem} \end {tikzpicture} </pre> <p style="text-align: center;">continued ...</p>



Table 11: ...continued

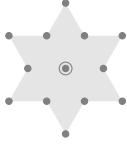
$G_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \wt [multiplicity=2]{0}{0} \end {rootSystem} \end {tikzpicture} </pre>
-------	---

Table 12: Weyl chambers

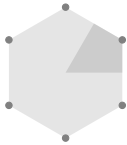

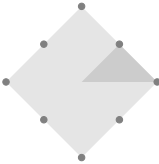
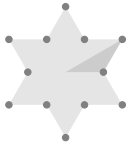
$A_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>
$B_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>
$C_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>
$G_2$	 <pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \WeylChamber \end {rootSystem} \end {tikzpicture} </pre>



Table 13: The positive root hyperplane

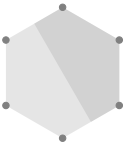

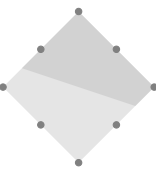



$A_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>
$B_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>
$C_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>
$G_2$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \positiveRootHyperplane \end {rootSystem} \end {tikzpicture} </pre>

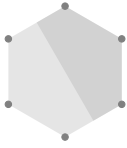



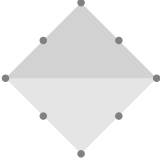
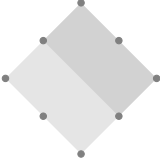
Table 14: Parabolic subgroups

$A_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre>
$A_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>

continued ...



Table 14: ...continued

$A_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>
$C_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre>
$C_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>

continued ...



Table 14: ...continued

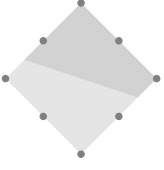


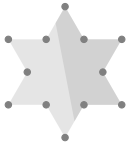
$C_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {1} \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {2} \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {3} \end {rootSystem} \end {tikzpicture} </pre>

Table 15: Parabolic subgroups with grading of the positive roots

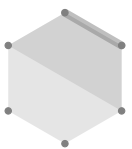
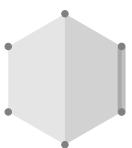
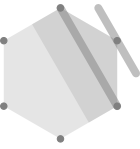



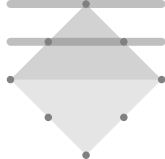
$A_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {1} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$A_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {2} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre> <p>continued ...</p>



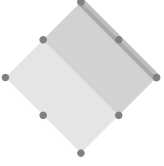
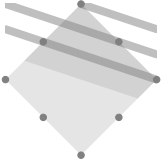
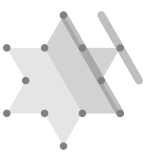


Table 15: ...continued

$A_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{A} \roots \parabolic {3} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {1} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {2} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$B_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{B} \roots \parabolic {3} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$C_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {1} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>

continued ...



Table 15: ...continued

$C_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {2} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$C_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{C} \roots \parabolic {3} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,1}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {1} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,2}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {2} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>
$G_{2,3}$		<pre> \begin {tikzpicture}[baseline=-.5] \begin {rootSystem}{G} \roots \parabolic {3} \parabolicgrading \end {rootSystem} \end {tikzpicture} </pre>

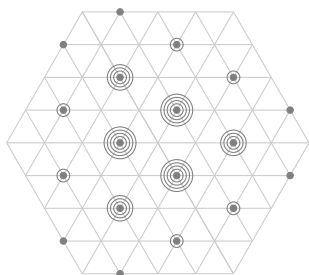
## 6. EXAMPLES OF WEIGHTS OF VARIOUS REPRESENTATIONS

Henceforth assume `\AutoSizeWeightLattice`true (the default).



Fulton and Harris, p. 186

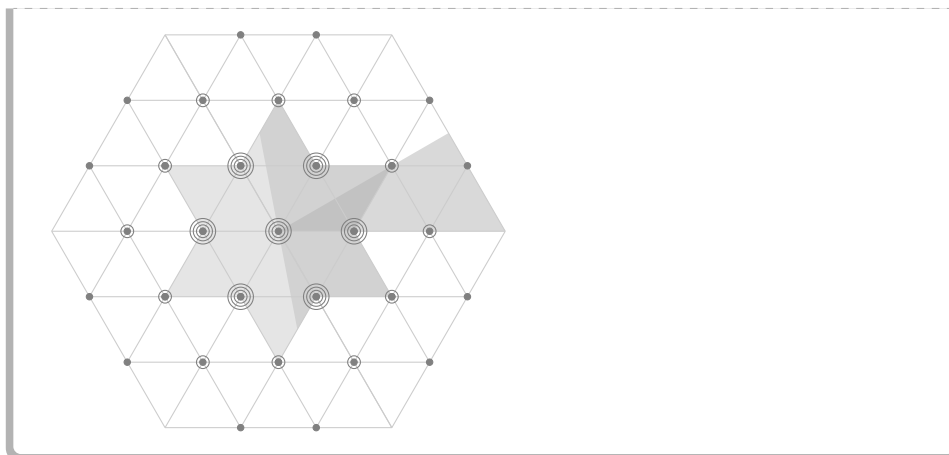
```
\begin{tikzpicture}
\begin{rootSystem}{A}
\foreach \x/\y/\m in
{0/ 1/5, -1/0/5, 1/-1/5, 2/ 0/4, -2/ 2/4, 0/-2/4,
 1/ 2/2, -1/3/2, 3/-2/2, 2/-3/2, -2/-1/2, -3/ 1/2,
 4/-1/1, 3/1/1, -3/ 4/1, -4/ 3/1, -1/-3/1, 1/-4/1}
{\wt[multiplicity=\m]{\x}{\y}}
\end{rootSystem}
\end{tikzpicture}
```



A representation of  $G_2$

```
\setlength\weightLength{1cm}
\begin{tikzpicture}
\begin{rootSystem}{G}
\roots
\foreach \m/\x/\y in {
1/1/1, 1/4/-1, 1/-1/2, 2/2/0, 1/5/-2,
2/0/1, 2/3/-1, 2/-2/2, 4/1/0, 1/-4/3,
2/4/-2, 4/-1/1, 4/2/-1, 2/-3/2, 1/5/-3,
4/0/0, 1/-5/3, 2/3/-2, 4/-2/1, 4/1/-1,
2/-4/2, 1/4/-3, 4/-1/0, 2/2/-2, 2/-3/1,
2/0/-1, 1/-5/2, 2/-2/0, 1/1/-2, 1/-4/1,
1/-1/-1}{\wt[multiplicity=\m]{\x}{\y}}
\positiveRootHyperplane
\WeylChamber
\end{rootSystem}
\end{tikzpicture}
```

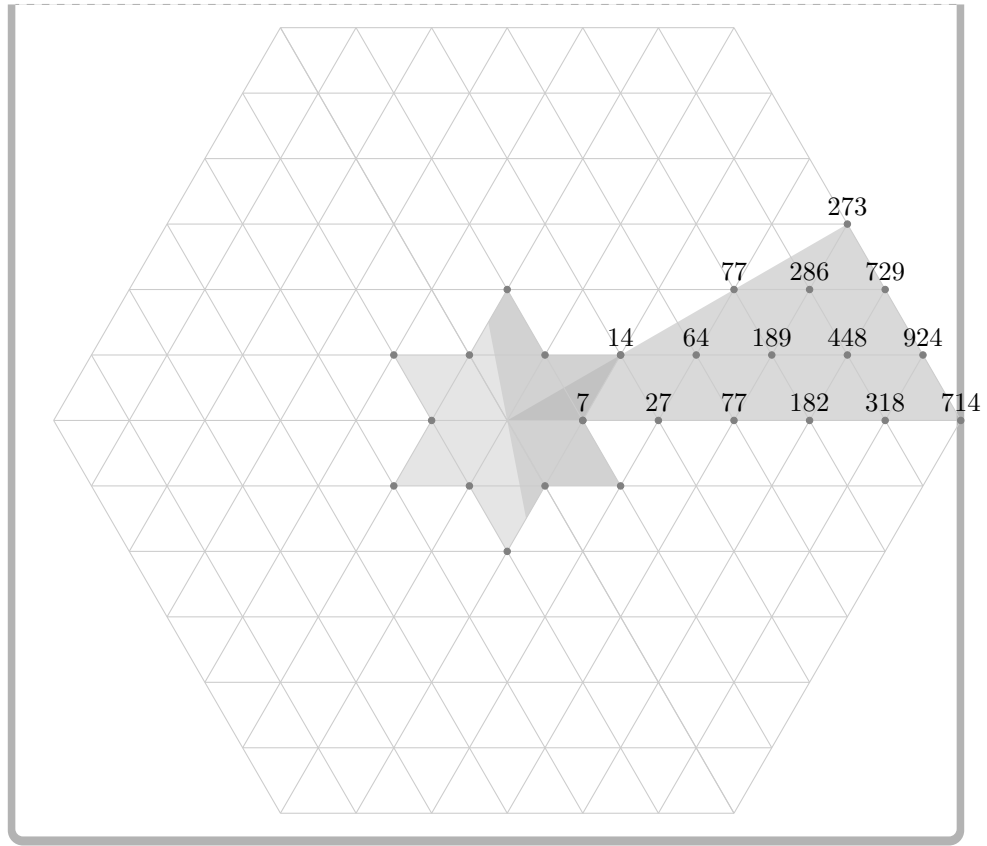




Dimensions of representations of  $G_2$ , parameterized by highest weight

```
\setlength\weightLength{1cm}
\begin{tikzpicture}
\begin{rootSystem}{G}
\roots
\foreach \x/\y/\d in {
0/1/14, 0/2/77, 0/3/273, 1/0/7, 1/1/64,
1/2/286, 2/0/27, 2/1/189, 2/2/729, 3/0/77,
4/0/182, 5/0/318, 6/0/714, 3/1/448, 4/1/924}
{\wt{\x}{\y}\node[black,above] at \weight{\x}{\y} {\(\d\)};}
\positiveRootHyperplane
\WeylChamber
\end{rootSystem}
\end{tikzpicture}
```







## 7. MORE OPTIONS

`\weightRadius`: length,  
 default = 1.2pt  
 Radius of dots used when marking specified weights.

`\weightLength`: length,  
 default = .5cm  
 Minimum distance between distinct weights.

`\gradingDot`: length,  
 default = 2pt  
 Size of dot around a root using to indicate a grading of a parabolic subalgebra which only contains one root.

`weight lattice`: TikZ style data,  
 default = `gray!40`  
 Style for drawing weight lattice lines.

`root`: TikZ style data,  
 default = `gray`  
 Style for drawing roots.

`root polygon`: TikZ style data,  
 default = `gray!40,opacity=.5`  
 Style for drawing a polygon which indicates the locations of the roots.

`hyperplane`: TikZ style data,  
 default = `gray!50,fill opacity=.5`  
 Style for drawing a hyperplane in a root system which contains either the positive roots, or (more generally) the positive height roots of a parabolic subgroup.

`Weyl chamber`: TikZ style data,  
 default = `gray!60,fill opacity=.5`  
 Style for drawing a wedge indicating the Weyl chamber of a root system.

`grading`: TikZ style data,  
 default = `line width=3pt,gray,opacity=0.5,line cap=round`  
 Style for drawing a thick line over top of some roots to indicate that they lie in the same grading associated to a parabolic subgroup.

```

\newlength\weightRadius
\setlength\weightRadius{1.2pt}
\newlength\weightLength
\setlength\weightLength{.5cm}
\newlength\gradingDot
\setlength\gradingDot{2pt}
\tikzstyle{weight lattice}=[gray!40]
\tikzstyle{root}=[gray]
\tikzstyle{root polygon}=[gray!40,opacity=.5]
\tikzstyle{hyperplane}=[gray!50,fill opacity=.5]
\tikzstyle{Weyl chamber}=[gray!60,fill opacity=.5]
\tikzstyle{grading}=[line width=3pt,gray,opacity=0.5,line cap=round]

```



## REFERENCES

1. R. W. Carter, *Lie algebras of finite and affine type*, Cambridge Studies in Advanced Mathematics, vol. 96, Cambridge University Press, Cambridge, 2005. MR 2188930
2. William Fulton and Joe Harris, *Representation theory*, Graduate Texts in Mathematics, vol. 129, Springer-Verlag, New York, 1991, A first course, Readings in Mathematics. MR 1153249

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