exactci: Exact Conditional Tests and Matching Confidence Intervals for binomial and Poisson variables

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1 Motivating Example for Package

Here is a problem which motivates this package. Consider comparing the difference between two exact Poisson rates. Suppose the observed rates for the two groups are 2/17877 and 10/20000. Since the counts are low, an exact test would be appropriate. In the stats package (R version 4.3.1 (2023-06-16 ucrt)), we could perform an exact test of the difference in Poisson rates by:

```
> poisson.test(c(2,10),c(17877,20000))

        Comparison of Poisson rates

data:  c(2, 10) time base: c(17877, 20000)
count1 = 2, expected count1 = 5.6637, p-value = 0.04213
alternative hypothesis: true rate ratio is not equal to 1
95 percent confidence interval:
 0.02383738 1.04995468
sample estimates:
rate ratio
 0.2237512
```

In Version 2.10.1, the p-value is significant at the 0.05 level ($p = 0.042$), but the 95 percent confidence interval contains the null rate ratio of 1, $(0.024, 1.05)$. That is because the p-value uses one method for defining significance and the confidence interval uses another method. In the 'exactci' package, the p-values and confidence intervals are always derived from the same p-value function (the p-value function has also been called the confidence curve, or the evidence function (see Hirji, 2006)).

There are three methods for defining the two-sided p-values in this package given by the 'tsmethod' option:

central: is 2 times the minimum of the one-sided p-values bounded above by 1. The name 'central' is motivated by the associated inversion confidence intervals which are central intervals, i.e., they guarantee that the true parameter has less than $\alpha/2$ probability of being less (more) than the lower (upper) tail of the $100(1-\alpha)\%$ confidence interval. This is called the TST (twice the smaller tail method) by Hirji (2006).

minlike: is the sum of probabilities of outcomes with likelihoods less than or equal to the observed likelihood. This is called the PB (probability based) method by Hirji (2006).

blaker: combines the probability of the smaller observed tail with the smallest probability of the opposite tail that does not exceed that observed tail probability. The name

'blaker' is motivated by Blaker (2000) which comprehensively studies the associated method for confidence intervals. This is called the CT (combined tail) method by Hirji (2006).

With the 'exactci' package we get automatic matching confidence intervals:

```
> library(exactci)
> poisson.exact(c(2,10),c(17877,20000),tsmethod="central")

        Exact two-sided Poisson test (central method)

data:  c(2, 10) time base: c(17877, 20000)
count1 = 2, expected count1 = 5.6637, p-value = 0.06056
alternative hypothesis: true rate ratio is not equal to 1
95 percent confidence interval:
 0.02383738 1.04995468
sample estimates:
rate ratio
 0.2237512


> poisson.exact(c(2,10),c(17877,20000),tsmethod="minlike")

        Exact two-sided Poisson test (sum of minimum likelihood method)

data:  c(2, 10) time base: c(17877, 20000)
count1 = 2, expected count1 = 5.6637, p-value = 0.04213
alternative hypothesis: true rate ratio is not equal to 1
95 percent confidence interval:
 0.03519552 0.94194758
sample estimates:
rate ratio
 0.2237512


> poisson.exact(c(2,10),c(17877,20000),tsmethod="blaker")

        Exact two-sided Poisson test (Blaker's method)

data:  c(2, 10) time base: c(17877, 20000)
count1 = 2, expected count1 = 5.6637, p-value = 0.04213
alternative hypothesis: true rate ratio is not equal to 1
95 percent confidence interval:
 0.03519552 0.93626967
sample estimates:
rate ratio
 0.2237512
```

Similar examples apply to exact binomial tests. For details see Fay (2010).

References

Blaker, H. (2000). "Confidence curves and improved exact confidence intervals for discrete distributions" *Canadian Journal of Statistics* **28,** 783-798 (correction **29,** 681).

Fay, M.P. (2010). "Two-sided Exact Tests and Matching Confidence Intervals for Discrete Data" . R Journal 2(1): 53-58 (available at `http://journal.r-project.org/archive/2010-1/RJou`

Hirji, K.F. (2006). *Exact Analysis of Discrete Data.* Chapman and Hall/CRC: New York.