# Package 'outlierMBC'

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backtrack

Move backwards from the minimum to a more conservative solution.

### Description

Given a vector of dissimilarity values, each corresponding to a different number of outliers, this function first finds the index and value of the minimum dissimilarity, then moves backwards from right to left to a reasonable solution with a lower index (i.e. lower number of outliers). Limits are placed on the maximum increase in dissimilarity from a single step (max\_step\_rise) and from all steps (max\_total\_rise), where both are defined in proportion to the minimum dissimilarity value.

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### Usage

```
backtrack(x, max_total_rise = 0.1, max_step_rise = 0.05)
```

### **Arguments**

### Value

backtrack returns a list with two elements, minimum and backtrack:

minimum is a list with the following elements: ind Index of the minimum solution.

val Value of the minimum solution.

backtrack is a list with the following elements: ind Index of the backtrack solution.

val Value of the backtrack solution.

### **Examples**

```
ombc_gmm_k3n1000o10 <-
   ombc_gmm(gmm_k3n1000o10[, 1:2], comp_num = 3, max_out = 20)
backtrack(ombc_gmm_k3n1000o10$distrib_diff_vec)</pre>
```

backtrack\_gmm

Fit a Gaussian mixture model to the backtrack solution.

#### **Description**

The backtrack function determines the number of outliers for the backtrack solution and plot\_backtrack plots this on a dissimilarity curve. backtrack\_gmm fits the mixture model corresponding to the number of outliers selected by the backtrack solution (or any manually specified number of outliers).

### Usage

```
backtrack_gmm(
    x,
    ombc_out,
    max_total_rise = 0.1,
    max_step_rise = 0.05,
    init_model = NULL,
    init_z = NULL,
    manual_outlier_num = NULL,
    verbose = TRUE
)
```

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### **Arguments**

Data. Χ ombc\_out An "outliermbc\_gmm" or "outliermbc\_lcwm" object, i.e. an output from ombc\_gmm or ombc\_lcwm. max\_total\_rise Upper limit for the cumulative increase, as a proportion of the global minimum dissimilarity, from all backward steps. Upper limit for the increase, as a proportion of the global minimum dissimilarity, max\_step\_rise from each backward step. Initial mixture model (mixture::gpcm best\_model). init\_model init\_z Initial component assignment probability matrix. manual\_outlier\_num User-specified number of outliers. verbose Whether the iteration count is printed.

#### Value

backtrack\_gmm returns a list with the following elements:

labels Vector of mixture component labels with outliers denoted by 0.

outlier\_bool Logical vector indicating if an observation has been classified as an outlier.

outlier\_num Number of observations classified as outliers.

mix Output from mixture::gpcm fitted to the non-outlier observations.

call Arguments / parameter values used in this function call.

### **Examples**

```
ombc_gmm_k3n1000o10 <- ombc_gmm(
   gmm_k3n1000o10[, 1:2],
   comp_num = 3, max_out = 20
)
backtrack_gmm(gmm_k3n1000o10[, 1:2], ombc_gmm_k3n1000o10)</pre>
```

backtrack\_lcwm

Fit a linear cluster-weighted model to the backtrack solution.

### **Description**

The backtrack function determines the number of outliers for the backtrack solution and plot\_backtrack plots this on a dissimilarity curve. backtrack\_gmm fits the mixture model corresponding to the number of outliers selected by the backtrack solution (or any manually specified number of outliers).

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### Usage

```
backtrack_lcwm(
   xy,
   x,
   ombc_lcwm_out,
   max_total_rise = 0.1,
   max_step_rise = 0.05,
   init_z = NULL,
   manual_outlier_num = NULL,
   verbose = TRUE
)
```

### **Arguments**

data. frame containing covariates and response. ху Covariate data only. Х An "outliermbc\_lcwm" object outputted by ombc\_lcwm. ombc\_lcwm\_out max\_total\_rise Upper limit for the cumulative increase, as a proportion of the global minimum dissimilarity, from all backward steps. Upper limit for the increase, as a proportion of the global minimum dissimilarity, max\_step\_rise from each backward step. init\_z Initial component assignment probability matrix. manual\_outlier\_num User-specified number of outliers. verbose Whether the iteration count is printed.

### Value

backtrack\_gmm returns a list with the following elements:

labels Vector of component labels with outliers denoted by 0.

outlier\_bool Logical vector indicating if an observation has been classified as an outlier.

outlier\_num Number of observations classified as outliers.

1cwm Output from flexCWM::cwm fitted to the non-outlier observations.

call Arguments / parameter values used in this function call.

### **Examples**

```
gross_lcwm_k3n1000010 <- find_gross(lcwm_k3n1000010, 20)

ombc_lcwm_k3n1000010 <- ombc_lcwm(
    xy = lcwm_k3n1000010[, c("X1", "Y")],
    x = lcwm_k3n1000010$X1,
    y_formula = Y ~ X1,
    comp_num = 2,
    max_out = 20,</pre>
```

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```
mnames = "V",
  gross_outs = gross_lcwm_k3n1000o10$gross_bool
)

backtrack_lcwm_k3n1000o10 <- backtrack_lcwm(
  xy = lcwm_k3n1000o10[, c("X1", "Y")],
  x = lcwm_k3n1000o10$X1,
  ombc_lcwm_out = ombc_lcwm_k3n1000o10
)</pre>
```

distrib\_diff\_gmm

Compute the dissimilarity for a Gaussian mixture model and identify the lowest density observation.

### **Description**

At each iteration of ombc\_gmm, distrib\_diff\_gmm computes the dissimilarity value of the current Gaussian mixture model. It also identifies the observation with the lowest mixture density.

#### **Usage**

```
distrib_diff_gmm(x, z, prop, mu, sigma, logdet)
```

### **Arguments**

X	Data.
Z	Component assignment probability matrix.
prop	Vector of component proportions.
mu	List of component mean vectors.
sigma	List of component covariance matrices.
logdet	Vector of log-determinants for covariance matrices.

### Value

```
distrib_diff_gmm returns a list with the following elements:

distrib_diff Aggregated dissimilarity across components.

distrib_diff_vec Vector containing dissimilarity value for each component.

choice_id Index of observation with lowest mixture density.

removal_dens Value of the lowest mixture density.
```

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distrib_diff_lcwm	Compute the dissimilarity for a linear cluster-weighted model and identify the lowest density observation.

### Description

At each iteration of ombc\_lcwm, distrib\_diff\_lcwm computes the dissimilarity value of the current linear cluster-weighted model. It also identifies the observation with the lowest mixture density.

### Usage

```
distrib_diff_lcwm(x, z, prop, mu, sigma, mod_list, y_sigma, dd_weight = 0.5)
```

### **Arguments**

x	Covariate data only.
z	Component assignment probability matrix.
prop	Vector of component proportions.
mu	Matrix of component mean vectors.
sigma	Array of component covariance matrices.
mod_list	List of component regression models.
y_sigma	Vector of component regression standard deviations.
dd_weight	A value between 0 and 1 which controls the weighting of the response and covariate dissimilarities when aggregating.

### Value

distrib\_diff\_lcwm\_lcwm returns a list with the following elements:

distrib\_diff Aggregated dissimilarity across components.

distrib\_diff\_vec Vector containing dissimilarity value for each component.

choice\_id Index of observation with lowest mixture density.

removal\_dens Value of the lowest mixture density.

distrib\_diff\_mat Two-column matrix containing response and covariate dissimilarities across components.

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distrib\_diff\_lcwm\_g Compute the dissimilarity for a single component of a Linear CWM.

### **Description**

Computes the covariate dissimilarity value, the response dissimilarity value, and their aggregated dissimilarity value. It also obtains the covariate, response, and joint densities for every observation.

### Usage

```
distrib_diff_lcwm_g(x, z_g, mu_g, sigma_g, mod_g, y_sigma_g, dd_weight = 0.5)
```

### **Arguments**

X	Covariate data only.
z_g	Component assignment probability vector.
mu_g	Component mean vector for the covariates.
sigma_g	Component covariance matrix for the covariates.
mod_g	Component regression model.
y_sigma_g	Component regression standard deviation for the response.
dd_weight	A value between 0 and 1 which controls the weighting of the response and covariate dissimilarities when aggregating.

### Value

distrib\_diff\_lcwm\_lcwm\_g returns a list with the following elements:

diff Aggregated dissimilarity value for this component.

dens Joint (covariate & response) density of all observations for this component.

diff\_x Covariate dissimilarity value for this component.

diff\_y Response dissimilarity value for this component.

dens\_x Covariate density of all observations for this component.

dens\_y Response density of all observations for this component.

distrib\_diff\_mahalanobis

Compute the dissimilarity for a single multivariate Gaussian distribution.

### **Description**

Compute the dissimilarity value and observation densities for a single multivariate Gaussian distribution. This could be a whole component in a Gaussian mixture model or the covariate part of a component in a Linear CWM.

### Usage

```
distrib_diff_mahalanobis(x, z_g, mu_g, sigma_g, logdet_g)
```

### **Arguments**

X	Data.
z_g	Assignment probability vector for component g.
mu_g	Mean vector for component g.
sigma_g	Covariance matrix for component g.
logdet_g	Log-determinants of covariance matrix for component g.

#### Value

distrib\_diff\_mahalanobis returns a list with the following elements:

diff Dissimilarity value for this component.

dens Gaussian density of all observations for this component.

mahalas Scaled squared sample Mahalanobis distances for all observations with respect to this component.

distrib\_diff\_residual Compute the response dissimilarity for a single component of a Linear CWM.

### **Description**

Computes the response dissimilarity value and the response density for every observation.

### Usage

```
distrib_diff_residual(x, z_g, mod_g, y_sigma_g)
```

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### Arguments

X	Covariate data only.
z_g	Component assignment probability vector.
mod_g	Component regression model.
y_sigma_g	Component regression standard deviation for the response.

### Value

```
distrib_diff_lcwm_residual returns a list with the following elements:
diff Response dissimilarity value for this component.
dens Response density of all observations for this component.
```

find\_gross

Find gross outliers.

### Description

The distance of each observation to its  $k^{th}$  nearest neighbour is computed. We assume that the largest max\_out kNN distances correspond to potential outliers. We select the next largest kNN distance, outside of the top max\_out, as a benchmark value. We multiply this benchmark kNN distance by multiplier to get the minimum threshold for our gross outliers. In other words, a gross outlier must have a kNN distance at least multiplier times greater than all of the observations which we do not consider to be potential outliers.

### Usage

```
find_gross(
    x,
    max_out,
    multiplier = 3,
    k_neighbours = floor(nrow(x)/100),
    manual_threshold = NULL,
    scale = TRUE
)
```

#### **Arguments**

x Data.

max\_out Maximum number of outliers.

multiplier Multiplicative factor used to get gross outlier threshold.

k\_neighbours Number of neighbours for dbscan::kNNdist.

manual\_threshold

Optional preset threshold.

scale Logical value controlling whether we apply scale to x.

get\_init\_z

### Value

```
find_gross returns a list with the following elements:
```

gross\_scatter ggplot of all kNN distances in index order.

```
gross_choice A numeric value indicating the elbow's location.

gross_bool A logical vector identifying the gross outliers.

gross_curve ggplot of the highest 2 * max_out kNN distances in decreasing order.
```

get\_init\_z

Obtain an initial clustering as a component assignment matrix.

### **Description**

Implement the specified initial clustering, either hierarchical clustering or k-means++, and return a binary component assignment matrix.

### Usage

```
get_init_z(
  comp_num,
  dist_mat = NULL,
  x = NULL,
  init_method = c("hc", "kmpp"),
  kmpp_seed = NULL
)
```

### Arguments

comp\_num Number of mixture components.

dist\_mat Euclidean distance matrix.

x Data.

init\_method Method used to initialise each mixture model.

kmpp\_seed Optional seed for k-means++ initialisation.

### Value

A component assignment matrix for initialisation.

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	Simulated data set consisting of 1000 observations from 3 Gaussian components and 10 outliers.
--	--

### **Description**

This data set was simulated using  $simulate\_gmm$ . There are 500 observations in Component 1, 250 observations in Component 2, and 250 observations in Component 3

### Usage

```
gmm_k3n1000o10
```

#### **Format**

```
gmm_k3n1000o10:
```

A data frame with 1010 rows and 3 columns:

X1, X2 Continuous variables.

**G** Component label: 0 for outliers; 1, 2, or 3 for true points.

#### **Source**

For simulation code, see gmm\_k3n1000o10.R in data-raw folder at https://github.com/UltanPDoherty/outlierMBC.

gmm_k3n2000o20	Simulated data set consisting of 2000 observations from 3 Gaussian
	components and 20 outliers.

### Description

This data set was simulated using simulate\_gmm. There are 1000 observations in Component 1, 500 observations in Component 2, and 500 observations in Component 3.

### Usage

```
gmm_k3n2000o20
```

### **Format**

```
gmm_k3n2000o20:
```

A data frame with 2020 rows and 3 columns:

X1, X2 Continuous variables.

**G** Component label: 0 for outliers; 1, 2, or 3 for true points.

gmm\_k3n4000o40

### **Source**

For simulation code, see gmm\_k3n2000o20.R in data-raw folder at https://github.com/UltanPDoherty/outlierMBC.

_	ated data set consisting of 4000 observations from 3 Gaussian onents and 40 outliers.
---	---

### Description

This data set was simulated using simulate\_gmm. There are 2000 observations in Component 1, 1000 observations in Component 2, and 1000 observations in Component 3.

### Usage

gmm\_k3n4000o40

### **Format**

gmm\_k3n4000o40:

A data frame with 4040 rows and 3 columns:

X1, X2 Continuous variables.

**G** Component label: 0 for outliers; 1, 2, or 3 for true points.

### **Source**

For simulation code, see gmm\_k3n4000o40.R in data-raw folder at https://github.com/UltanPDoherty/outlierMBC.

lcwm_k3n1000o10	Simulated data set consisting of 1000 observations from 3 Gaussian components and 10 outliers.
	components and 10 outliers.

### **Description**

This data set was simulated using  $simulate_lcwm$ . There are 300 observations in Component 1, 300 observations in Component 2, and 400 observations in Component 3

### Usage

lcwm\_k3n1000o10

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### **Format**

1cwm\_k3n1000o10:

A data frame with 1010 rows and 3 columns:

X1 Continuous explanatory variable.

Y Continuous response variable.

**G** Component label: 0 for outliers; 1, 2, or 3 for true points.

#### **Source**

For simulation code, see lcwm\_k3n1000o10.R in data-raw folder at https://github.com/UltanPDoherty/outlierMBC.

1cwm\_k3n2000o20 Simulated data set consisting of 2000 observations from 3 Gaussian components and 20 outliers.

### **Description**

This data set was simulated using simulate\_lcwm. There are 600 observations in Component 1, 600 observations in Component 2, and 800 observations in Component 3.

### Usage

1cwm\_k3n2000o20

#### **Format**

1cwm\_k3n2000o20:

A data frame with 2020 rows and 3 columns:

X1 Continuous explanatory variable.

Y Continuous response variable.

**G** Component label: 0 for outliers; 1, 2, or 3 for true points.

### **Source**

For simulation code, see lcwm\_k3n2000o20.R in data-raw folder at https://github.com/UltanPDoherty/outlierMBC.

lcwm\_k3n4000o40

lcwm_k3n4000o40	Simulated data set consisting of 4000 observations from 3 Gaussian components and 40 outliers.
	-

### **Description**

This data set was simulated using simulate\_lcwm. There are 1200 observations in Component 1, 1200 observations in Component 2, and 1600 observations in Component 3.

### Usage

lcwm\_k3n4000o40

### **Format**

1cwm k3n4000o40:

A data frame with 4040 rows and 3 columns:

**X1** Continuous explanatory variable.

Y Continuous response variable.

**G** Component label: 0 for outliers; 1, 2, or 3 for true points.

#### Source

For simulation code, see lcwm\_k3n4000o40.R in data-raw folder at https://github.com/UltanPDoherty/outlierMBC.

### **Description**

Constructor for "outliermbc\_gmm" S3 class.

### Usage

```
new_outliermbc_gmm(x = list())
```

### **Arguments**

x List.

#### Value

"outliermbc\_gmm" S3 object.

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new\_outliermbc\_lcwm

Constructor for "outliermbc\_lcwm" S3 object.

### Description

Constructor for "outliermbc\_lcwm" S3 object.

### Usage

```
new_outliermbc_lcwm(x = list())
```

### **Arguments**

Х

List.

### Value

"outliermbc\_lcwm" S3 object.

ombc\_gmm

Sequentially identify outliers while fitting a Gaussian mixture model.

### Description

This function performs model-based clustering and outlier identification. It does so by iteratively fitting a Gaussian mixture model and removing the observation that is least likely under the model. Its procedure is summarised below:

- 1. Fit a Gaussian mixture model to the data.
- 2. Compute a dissimilarity between the theoretical and observed distributions of the scaled squared sample Mahalanobis distances for each mixture component.
- 3. Aggregate across the components to obtain a single dissimilarity value.
- 4. Remove the observation with the lowest mixture density.
- 5. Repeat Steps 1-4 until max\_out observations have been removed.
- 6. Identify the number of outliers which minimised the aggregated dissimilarity, remove only those observations, and fit a Gaussian mixture model to the remaining data.

ombc\_gmm

### Usage

```
ombc_gmm(
  х,
  comp_num,
 max_out,
 gross_outs = rep(FALSE, nrow(x)),
  init_scheme = c("update", "reinit", "reuse"),
 mnames = "VVV",
  nmax = 1000,
  atol = 1e-08,
  init_z = NULL,
  init_model = NULL,
  init_method = c("hc", "kmpp"),
  init_scaling = FALSE,
  kmpp\_seed = 123,
  fixed_labels = NULL,
  verbose = TRUE
)
```

### **Arguments**

Data.
Number of mixture components.
Maximum number of outliers.
Logical vector identifying gross outliers.
Which initialisation scheme to use.
Model names for mixture::gpcm.
Maximum number of iterations for mixture::gpcm.
EM convergence tolerance threshold for mixture::gpcm.
Initial component assignment probability matrix.
<pre>Initial mixture model (mixture::gpcm best_model).</pre>
Method used to initialise each mixture model.
Logical value controlling whether the data should be scaled for initialisation.
Optional seed for k-means++ initialisation.
Cluster labels that are known a prior. See label argument in mixture::gpcm.
Whether the iteration count is printed.

### Value

ombc\_gmm returns an object of class "outliermbc\_gmm", which is essentially a list with the following elements:

labels Vector of mixture component labels with outliers denoted by 0. outlier\_bool Logical vector indicating if an observation has been classified as an outlier.

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outlier\_num Number of observations classified as outliers.

outlier\_rank Order in which observations are removed from the data set. Observations which were provisionally removed, including those that were eventually not classified as outliers, are ranked from 1 to max\_out. All gross outliers have rank 1. If there are gross\_num gross outliers, then the observations removed during the main algorithm itself will be numbered from gross\_num + 1 to max\_out. Observations that were ever removed have rank 0.

gross\_outs Logical vector identifying the gross outliers. This is identical to the gross\_outs vector passed to this function as an argument / input.

mix Output from mixture::gpcm fitted to the non-outlier observations.

loglike Vector of log-likelihood values for each iteration.

removal\_dens Vector of mixture densities for the removed observations. These are the lowest mixture densities at each iteration.

distrib\_diff\_vec Vector of aggregated cross-component dissimilarity values for each iteration.

distrib\_diff\_mat Matrix of component-specific dissimilarity values for each iteration.

call Arguments / parameter values used in this function call.

version Version of outlierMBC used in this function call.

conv\_status Logical vector indicating which iterations' mixture models reached convergence during model-fitting.

### **Examples**

```
ombc_gmm_k3n1000o10 <- ombc_gmm(
   gmm_k3n1000o10[, 1:2],
   comp_num = 3, max_out = 20
)
plot_curve(ombc_gmm_k3n1000o10)</pre>
```

ombc\_1cwm

Sequentially identify outliers while fitting a linear cluster-weighted model.

### **Description**

This function performs model-based clustering, clusterwise regression, and outlier identification. It does so by iteratively fitting a linear cluster-weighted model and removing the observation that is least likely under the model. Its procedure is summarised below:

- 1. Fit a linear cluster-weighted model to the data.
- 2. Compute a dissimilarity between the theoretical and observed distributions of the scaled squared sample Mahalanobis distances for each mixture component.
- 3. Compute a dissimilarity between the theoretical and observed distributions of the scaled squared studentised residuals for each mixture component.
- 4. Aggregate these two dissimilarities to obtain one dissimilarity value for each component.

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- 5. Aggregate across the components to obtain a single dissimilarity value.
- 6. Remove the observation with the lowest mixture density.
- 7. Repeat Steps 1-6 until max\_out observations have been removed.
- 8. Identify the number of outliers which minimised the aggregated dissimilarity, remove only those observations, and fit a linear cluster-weighted model to the remaining data.

### Usage

```
ombc_lcwm(
 хy,
 х,
 y_formula,
  comp_num,
 max_out,
 gross_outs = rep(FALSE, nrow(x)),
 init_scheme = c("update", "reinit", "reuse"),
 mnames = "VVV",
 nmax = 1000,
 atol = 1e-08,
 init_z = NULL,
  init_method = c("hc", "kmpp"),
  init_scaling = TRUE,
 kmpp\_seed = 123,
 verbose = TRUE,
  dd_weight = 0.5
)
```

### **Arguments**

ху	data.frame containing covariates and response.
X	Covariate data only.
y_formula	Regression formula.
comp_num	Number of mixture components.
max_out	Maximum number of outliers.
gross_outs	Logical vector identifying gross outliers.
init_scheme	Which initialisation scheme to use.
mnames	Model names for mixture::gpcm.
nmax	Maximum number of iterations for flexCWM::cwm.
atol	EM convergence threshold for flexCWM::cwm.
init_z	Initial component assignment probability matrix.
init_method	Method used to initialise each mixture model.
init_scaling	Logical value controlling whether the data should be scaled for initialisation.
kmpp_seed	Optional seed for k-means++ initialisation.
verbose	Whether the iteration count is printed.
dd_weight	A value between 0 and 1 which controls the weighting of the response and covariate dissimilarities when aggregating.

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#### Value

ombc\_lcwm returns an object of class "outliermbc\_lcwm", which is essentially a list with the following elements:

labels Vector of mixture component labels with outliers denoted by 0.

outlier\_bool Logical vector indicating if an observation has been classified as an outlier.

outlier\_num Number of observations classified as outliers.

outlier\_rank Order in which observations are removed from the data set. Observations which were provisionally removed, including those that were eventually not classified as outliers, are ranked from 1 to max\_out. All gross outliers have rank 1. If there are gross\_num gross outliers, then the observations removed during the main algorithm itself will be numbered from gross\_num + 1 to max\_out. Observations that were ever removed have rank 0.

gross\_outs Logical vector identifying the gross outliers. This is identical to the gross\_outs vector passed to this function as an argument / input.

1cwm Output from flexCWM::cwm fitted to the non-outlier observations.

loglike Vector of log-likelihood values for each iteration.

removal\_dens Vector of mixture densities for the removed observations. These are the lowest mixture densities at each iteration.

distrib\_diff\_vec Vector of aggregated cross-component dissimilarity values for each iteration.

distrib\_diff\_mat Matrix of component-specific dissimilarity values for each iteration.

distrib\_diff\_arr Array of component-specific response and covariate dissimilarity values for each iteration.

call Arguments / parameter values used in this function call.

version Version of outlierMBC used in this function call.

conv\_status Logical vector indicating which iterations' mixture models reached convergence during model-fitting.

#### **Examples**

```
gross_lcwm_k3n1000o10 <- find_gross(lcwm_k3n1000o10, 20)

ombc_lcwm_k3n1000o10 <- ombc_lcwm(
    xy = lcwm_k3n1000o10[, c("X1", "Y")],
    x = lcwm_k3n1000o10$X1,
    y_formula = Y ~ X1,
    comp_num = 3,
    max_out = 20,
    mnames = "V",
    gross_outs = gross_lcwm_k3n1000o10$gross_bool
)</pre>
```

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```
plot.outliermbc_gmm plot method for "outliermbc_gmm" S3 class.
```

### Description

```
plot method for "outliermbc_gmm" S3 class.
```

### Usage

```
## S3 method for class 'outliermbc_gmm'
plot(x, backtrack = FALSE, ...)
```

### Arguments

```
x Listbacktrack Logical
```

... Other arguments

### Value

A ggplot

```
\verb|plot.outliermbc_lcwm|| \textit{plot method for "outliermbc_lcwm" S3 class.}
```

### **Description**

```
plot method for "outliermbc_lcwm" S3 class.
```

### Usage

```
## S3 method for class 'outliermbc_lcwm'
plot(x, backtrack = FALSE, ...)
```

### **Arguments**

```
x List
backtrack Logical
```

... Other arguments

#### Value

A ggplot

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plot_backtrack	Plot the dissimilarity curve showing the backtrack solution.	
prot_backtrack	r tot the dissimilarity curve showing the backtrack solution.	

### Description

Plots a rescaled dissimilarity curve where the dissimilarity values (y axis) have been divided by their minimum so that the rescaled minimum is at 1. Vertical lines mark the minimum and backtrack solutions.

### Usage

```
plot_backtrack(ombc_out, max_total_rise = 0.1, max_step_rise = 0.05)
```

### **Arguments**

ombc_out	An "outliermbc_gmm" or "outliermbc_lcwm" object, i.e. an output from ombc_gmm or ombc_lcwm.
max_total_rise	Upper limit for the cumulative increase, as a proportion of the global minimum dissimilarity, from all backward steps.
max_step_rise	Upper limit for the increase, as a proportion of the global minimum dissimilarity, from each backward step.

### Value

plot\_backtrack returns a ggplot of the rescaled dissimilarity curve showing the minimum solution and the backtrack solutions.

plot_comparison	Plot multiple dissimilarity curves.

### Description

Given a range of ombc\_gmm outputs, each arising from a different model, this function is designed to produce a graphical aid for selecting the best model. It displays the dissimilarity curves from each of these models on the same plot.

### Usage

```
plot_comparison(ombc_list)
```

### **Arguments**

ombc\_list A list of outputs from ombc\_gmm.

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### Value

plot\_comparison returns a ggplot object consisting of multiple dissimilarity curves overlaid on the same plot.

plot\_comparison\_bic

Plot multiple dissimilarity curves.

### Description

Given a range of ombc\_gmm outputs, each arising from a different model, this function is designed to produce a graphical aid for selecting the best model. It displays the dissimilarity curves from each of these models on the same plot.

### Usage

```
plot_comparison_bic(ombc_list)
```

### **Arguments**

ombc\_list

A list of outputs from ombc\_gmm.

#### Value

plot\_comparison returns a ggplot object consisting of multiple dissimilarity curves overlaid on the same plot.

plot\_curve

Plot the dissimilarity curve.

### **Description**

Given the output from ombc\_gmm or ombc\_lcwm, this function extracts the dissimilarity value associated with each outlier number and plots them as a curve. It also draws a vertical line at the outlier number which minimised the dissimilarity.

### Usage

```
plot_curve(ombc_out)
```

### **Arguments**

 $ombc\_out$ 

An "outliermbc\_gmm" or "outliermbc\_lcwm" object, i.e. an output from ombc\_gmm or ombc\_lcwm.

### Value

plot\_curve returns a ggplot object showing the dissimilarity values as a curve and marking the minimum solution with a vertical line.

print.outliermbc\_gmm

plot\_selection

Plot dissimilarity values for multiple solutions.

#### **Description**

Given a range of ombc\_gmm outputs, each arising from a different model, this function is designed to produce a graphical aid for selecting the best model. It plots the dissimilarity values of the models' minimum and backtrack solutions against their number of components (x\_axis = "comp\_num"), number of outliers (x\_axis = "outlier\_num"), or number of parameters (x\_axis = "param\_num").

### Usage

```
plot_selection(ombc_list, x_axis = c("comp_num", "outlier_num", "param_num"))
```

### Arguments

ombc\_list A list of outputs from ombc\_gmm.

x\_axis The quantity to be plotted on the x axis.

#### Value

plot\_selection return a ggplot object plotting the minimum dissimilarity and backtrack solutions from a number of outputs from ombc\_gmm versus their number of components, outliers, or parameters.

```
print.outliermbc_gmm print method for "outliermbc_gmm" S3 class.
```

### **Description**

```
print method for "outliermbc_gmm" S3 class.
```

### Usage

```
## S3 method for class 'outliermbc_gmm'
print(x, backtrack = FALSE, max_total_rise = 0.1, max_step_rise = 0.05, ...)
```

### **Arguments**

```
    x List
    backtrack Logical
    max_total_rise Upper limit for the cumulative increase, as a proportion of the global minimum dissimilarity, from all backward steps.
    max_step_rise Upper limit for the increase, as a proportion of the global minimum dissimilarity, from each backward step.
    ... Other arguments
```

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### Value

A ggplot

```
print.outliermbc_lcwm print method for "outliermbc_lcwm" S3 class.
```

### Description

```
print method for "outliermbc_lcwm" S3 class.
```

### Usage

```
## S3 method for class 'outliermbc_lcwm'
print(x, backtrack = FALSE, max_total_rise = 0.1, max_step_rise = 0.05, ...)
```

### Arguments

x	List
backtrack	Logical
max_total_rise	Upper limit for the cumulative increase, as a proportion of the global minimum dissimilarity, from all backward steps.
max_step_rise	Upper limit for the increase, as a proportion of the global minimum dissimilarity, from each backward step.
	Other arguments

### Value

A ggplot

simulate_gmm	Simulate data from a Gaussian mixture model with outliers.	

### Description

Simulates data from a Gaussian mixture model, then simulates outliers from a hyper-rectangle, with a rejection step to ensure that the outliers are sufficiently unlikely under the model.

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### Usage

```
simulate_gmm(
    n,
    mu,
    sigma,
    outlier_num,
    seed = NULL,
    crit_val = 0.9999,
    range_multiplier = 1.5,
    verbose = TRUE,
    max_rejection = 1e+06
)
```

### **Arguments**

n Vector of component sizes.

mu List of component mean vectors.

sigma List of component covariance matrices.

outlier\_num Desired number of outliers.

seed Seed.

range\_multiplier

How much greater should the range of the Uniform samples be than the range

of the Normal samples?

verbose Whether a message should be printed if a high number of outliers are being

simulated. This suggests that many simulated outliers are being rejected and the

other arguments may need to be adjusted.

max\_rejection Maximum number of simulated outliers to be rejected.

#### **Details**

The simulated outliers are sampled from a Uniform distribution over a hyper-rectangle. For each dimension, the hyper-rectangle is centred at the midpoint between the maximum and minimum values for that variable from all of the Gaussian observations. Its width in that dimension is the distance between the minimum and maximum values for that variable multiplied by the value of range\_multiplier. If range\_multiplier = 1, then this hyper-rectangle is the axis-aligned minimum bounding box for all of the Gaussian data points in this data set.

The crit\_val ensures that it would have been sufficiently unlikely for a simulated outlier to have been sampled from any of the Gaussian components. The Mahalanobis distances of a proposed outlier from each component's mean vector with respect to that component's covariance matrix are computed. If any of these Mahalanobis distances are smaller than the critical value of the appropriate Chi-squared distribution, then the proposed outlier is rejected. In summary, for a Uniform sample to be accepted, it must be sufficiently far from each component in terms of Mahalanobis distance.

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### Value

simulate\_gmm return a data.frame with continuous variables X1, X2, ..., followed by a mixture component label vector G with outliers denoted by  $\emptyset$ .

### **Examples**

```
gmm_k3n1000o10 <- simulate_gmm(
    n = c(500, 250, 250),
    mu = list(c(-1, 0), c(+1, -1), c(+1, +1)),
    sigma = list(diag(c(0.2, 4 * 0.2)), diag(c(0.2, 0.2)), diag(c(0.2, 0.2))),
    outlier_num = 10,
    seed = 123,
        crit_val = 0.9999,
        range_multiplier = 1.5
)

plot(
    gmm_k3n1000o10[, c("X1", "X2")],
    col = gmm_k3n1000o10$G + 1, pch = gmm_k3n1000o10$G + 1
)</pre>
```

simulate\_lcwm

Simulate data from a linear cluster-weighted model with outliers.

### **Description**

Simulates data from a linear cluster-weighted model, then simulates outliers from a region around each mixture component, with a rejection step to control how unlikely the outliers are under the model.

### Usage

```
simulate_lcwm(
    n,
    mu,
    sigma,
    beta,
    error_sd,
    outlier_num,
    outlier_type = c("x_and_y", "x_only", "y_only"),
    seed = NULL,
    prob_range = c(1e-08, 1e-06),
    range_multipliers = c(3, 3),
    more_extreme = FALSE
)
```

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#### **Arguments**

n Vector of component sizes.

mu List of component mean vectors.

sigma List of component covariance matrices.

beta List of component regression coefficient vectors.

error\_sd Vector of component regression error standard deivations.

outlier\_num Desired number of outliers.

outlier\_type Character string governing whether the outliers are outlying with respect to the

explanatory variable only ("x\_only"), the response variable only ("y\_only"),

or both ("x\_and\_y"). "x\_and\_y" is the default value.

seed Seed.

prob\_range Values for uniform sample rejection.

range\_multipliers

For every explanatory variable, the sampling region The sampling region for the Uniform distribution used to simulate proposed outliers is controlled by multi-

plying the component widths by these values.

more\_extreme Whether to return a column in the data frame consisting of the probabilities of

sampling more extreme true observations than the simulated outliers.

#### **Details**

simulate\_lcwm samples a user-defined number of outliers for each component. However, even though an outlier may be associated with one component, it must be outlying with respect to every component.

The covariate values of the simulated outliers for a given component g are sampled from a Uniform distribution over a hyper-rectangle which is specific to that component. For each covariate dimension, the hyper-rectangle is centred at the midpoint between the maximum and minimum values for that variable from all of the Gaussian observations from component g. Its width in that dimension is the distance between the minimum and maximum values for that variable multiplied by the value of range\_multiplier[1].

The response values of the simulated outliers for a given component g are obtained by sampling random errors from a Uniform distribution over a univariate interval, simulating covariate values as discussed above, computing the mean response value for those covariate values, then adding this simulated error to the response. The error sampling interval is centred at the midpoint between the maximum and minimum errors for that variable from all of the Gaussian observations from component g. Its width is the distance between the minimum and maximum errors multiplied by the value of range\_multiplier[2].

A proposed outlier for component g is rejected if the probability of sampling a more extreme point from any of the components is greater than prob\_range[2] or if the probability of sampling a less extreme point from component g is less than prob\_range[1]. This can be visualised as a pair of inner and outer envelopes around each component. To be accepted, a proposed outlier must lie inside the outer envelope for its component and outside the inner envelopes of all components. Setting prob\_range[1] = 0 will eliminate the outer envelope, while setting prob\_range[2] = 0 will eliminate the inner envelope.

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By setting outlier\_type = "x\_only" and giving arbitrary values to error\_sd (e.g. a zero vector) and beta (e.g. a list of zero vectors), then ignoring the simulated Y variable, simulate\_lcwm can be used to simulate a Gaussian mixture model. Since simulate\_lcwm simulates component-specific outliers from sampling regions around each component, rather than a single sampling region around all of the components, this will not be equivalent to simulate\_gmm. simulate\_lcwm also allows the user to set an upper bound on how unlikely an outlier is, as well as a lower bound, whereas simulate\_gmm only sets a lower bound.

#### Value

simulate\_1cwm returns a data.frame with continuous variables X1, X2, ..., followed by a continuous response variable, Y, and a mixture component label vector G with outliers denoted by 0. The optional variable more\_extreme may be included, if specified by the corresponding argument.

### **Examples**

```
lcwm_k3n1000o10 <- simulate_lcwm(</pre>
  n = c(300, 300, 400),
  mu = list(c(3), c(6), c(3)),
  sigma = list(as.matrix(1), as.matrix(0.1), as.matrix(1)),
  beta = list(c(0, 0), c(-75, 15), c(0, 5)),
  error_sd = c(1, 1, 1),
  outlier_num = c(3, 3, 4),
  outlier_type = "x_and_y",
  seed = 123,
  prob_range = c(1e-8, 1e-6),
  range_multipliers = c(1, 2)
)
plot(
  lcwm_k3n1000o10[, c("X1", "Y")],
  col = 1cwm_k3n1000o10\$G + 1,
  pch = 1cwm_k3n1000o10\$G + 1
)
```

test\_outlier\_ombc

Check if a new sample satisfies the outlier criteria.

### Description

This function checks whether a given sample is an acceptable outlier with respect to prob\_range and also computes the probability of sampling a more extreme point from component g.

### Usage

```
test_outlier_ombc(
  outlier_type,
  mu,
  sigma,
```

30 try\_mixture\_gpcm

```
beta,
  error_sd,
  x_sample,
  y_sample,
  prob_range,
  g
)
```

### **Arguments**

outlier\_type Character string governing whether the outliers are outlying with respect to the

explanatory variable only ("x\_only"), the response variable only ("y\_only"),

or both ("x\_and\_y"). "x\_and\_y" is the default value.

mu List of component mean vectors.

sigma List of component covariance matrices.

beta List of component regression coefficient vectors.

error\_sd Vector of component regression error standard deivations.

x\_sampleNew covariate sample.y\_sampleNew response sample.

prob\_range Values for uniform sample rejection.

g Component number.

#### Value

test\_outlier\_ombc returns a vector consisting of a logical value indicating whether the new sample satisfies the outlier checks, and a numeric value giving the probability of sampling a more extreme point from component g.

try\_mixture\_gpcm Run mixture::gpcm and try alternative covariance structures or ini-

tialisations if necessary.

### **Description**

If mixture::gpcm returns an error, this function first tries the other covariance structures, and then tries a k-means initialisation.

#### Usage

```
try_mixture_gpcm(x, comp_num, mnames, z, nmax, atol, fixed_labels)
```

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### Arguments

x Data.

comp\_num Number of mixture components.
mnames Model names for mixture::gpcm.

z Component assignment probability matrix for initialisation.

nmax Maximum number of iterations for mixture::gpcm.

atol EM convergence tolerance threshold for mixture::gpcm.

fixed\_labels Cluster labels that are known a prior. See label argument in mixture::gpcm.

#### Value

Object of class "gpcm" outputted by mixture::gpcm.

### **Description**

This function calls uniform\_sample\_lcwm to sample a proposed outlier and then calls test\_outlier\_ombc to check if it satisfies the required criteria.

### Usage

```
uniform_outlier_ombc(
  outlier_type,
  mu,
  sigma,
  beta,
  error_sd,
  g,
  uniform_spans,
  prob_range
)
```

### **Arguments**

outlier\_type Character string governing whether the outliers are outlying with respect to the

explanatory variable only ("x\_only"), the response variable only ("y\_only"),

or both ("x\_and\_y"). "x\_and\_y" is the default value.

mu List of component mean vectors.

sigma List of component covariance matrices.

beta List of component regression coefficient vectors.

error\_sd Vector of component regression error standard deivations.

g Component index.

uniform\_spans Covariate and response error spans.

prob\_range Values for uniform sample rejection.

### Value

uniform\_outlier\_ombc returns a simulated outlier as a vector containing its covariate values, response value, and its component label 0. This vector's final element is the probability of sampling a more extreme Gaussian point from this outlier's associated component.

```
uniform_sample_lcwm Sample a potential outlier.
```

### **Description**

If outlier\_type = "x\_and\_y", then both the covariate values and response error of the outlier proposed by this function will be Uniformly distributed. If outlier\_type = "x\_only", then the covariate values will be Uniformly distributed but the response error will be Normally distributed. If outlier\_type = "y\_only", then the response error will be Uniformly distributed but the covariate values will be Normally distributed.

### Usage

```
uniform_sample_lcwm(
  outlier_type,
  mu_g,
  sigma_g,
  beta_g,
  error_sd_g,
  uniform_spans_g
)
```

### Arguments

outlier_type	Character string governing whether the outliers are outlying with respect to the explanatory variable only ("x_only"), the response variable only ("y_only"), or both ("x_and_y"). "x_and_y" is the default value.
mu_g	Covariate mean vector for component g.
sigma_g	Covariate covariance matrix for component g.
beta_g	Regression coefficient vector for component g.
error_sd_g	Regression error standard deviation for component g.
uniform_spans_g	
	Covariate and response error ranges for component g.

### Value

uniform\_sample\_lcwm returns a list with the following elements:

- x Vector of covariate values.
- y Response value.

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uniform\_spans\_lcwm

Obtain the span of the observations for each component.

### **Description**

Determine the minimum and maximum values for each covariate / explanatory variable and for the response errors from all Gaussian observations.

### Usage

```
uniform_spans_lcwm(range_multipliers, covariates_g, errors_g)
```

### Arguments

range\_multipliers

For every explanatory variable, the sampling region The sampling region for the Uniform distribution used to simulate proposed outliers is controlled by multiplying the component widths by these values.

plying the component widths by these values.

covariates\_g Covariate values of the sampled observations.

errors\_g Response errors of the sampled observations.

### Value

uniform\_spans\_1cwm returns a 2-column matrix. The final row contains the minimum and maximum values of the response errors, while the previous rows contain the minimum and maximum values for each covariate.

```
validate_outliermbc_gmm
```

Validator for "outliermbc\_gmm" S3 class.

### Description

Validator for "outliermbc\_gmm" S3 class.

### Usage

```
validate_outliermbc_gmm(x)
```

### **Arguments**

Х

List.

```
validate\_outliermbc\_lcwm
```

Validator for "outliermbc\_lcwm" S3 class.

### Description

Validator for "outliermbc\_lcwm" S3 class.

### Usage

```
validate_outliermbc_lcwm(x)
```

### Arguments

x List.

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