

Package ‘reliacoef’

March 5, 2026

Type Package

Title Unidimensional and Multidimensional Reliability Coefficients

Version 1.0.1

Description Calculates and compares various reliability coefficients for unidimensional and multidimensional scales. Supported unidimensional estimators include coefficient alpha, congeneric reliability, the Gilmer-Feldt coefficient, Feldt's classical congeneric reliability, Hancock's H, Heise-Bohrnstedt's omega, Kaiser-Caffrey's alpha, and Ten Berge and Zegers' mu series. Multidimensional estimators include stratified alpha, maximal reliability, correlated factors reliability, second-order factor reliability, and bifactor reliability. See Cho (2021) <[doi:10.1007/s11336-021-09801-1](https://doi.org/10.1007/s11336-021-09801-1)>, Cho (2024) <[doi:10.1037/met0000475](https://doi.org/10.1037/met0000475)>, Cho (2025) <[doi:10.1037/met0000525](https://doi.org/10.1037/met0000525)>.

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Encoding UTF-8

LazyData true

RoxygenNote 7.3.3

Imports lavaan, psych, matrixcalc, stats

Suggests knitr, rmarkdown, Rcsdp, testthat (>= 3.0.0)

Depends R (>= 3.5.0)

VignetteBuilder knitr

NeedsCompilation no

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Repository CRAN

Date/Publication 2026-03-05 10:30:10 UTC

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alpha

Obtain Coefficient Alpha

Description

Alpha, also referred to as Cronbach's alpha or tau-equivalent reliability, is the most commonly used reliability coefficient.

Usage

```
alpha(x, standardized = FALSE)
```

Arguments

`x` a data frame of raw data or a covariance matrix
`standardized` If TRUE, the calculation is based on the correlation matrix.

Details

History: Kuder and Richardson (1937) first developed this formula, but they did not name it alpha. At the time, it was referred to as Kuder-Richardson Formula 20. Cronbach (1951) argued that this name was strange and insisted on calling it coefficient alpha, which is now widely used.

Interpretations: Alpha can be derived with an ANOVA approach to reliability (Hoyt 1941). Alpha is λ_3 , one of the six lower bounds of reliability (Guttman 1945). Alpha is the average of λ_4 values obtained over all possible split-halves (Cronbach 1951). Alpha equals reliability if the x meets the condition of being essentially tau-equivalent (Novick & Lewis, 1967). Alpha is μ_0 , the first in Ten Berge and Socan's (1978) series of reliability coefficients.

Accuracy: Alpha is found to be inferior in several studies examining the accuracy of the reliability coefficients (Cho 2024a, 2024b). Alpha can produce negative reliability estimates and is sensitive to the violation of the assumption of essential tau-equivalence (Cho 2021; Cho and Kim 2015).

Value

Coefficient alpha reliability estimate.

References

- Cho, E. (2021). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*, 86(4), 877-886.
- Cho, E. (2024a). Beyond alpha and omega: The accuracy of single-test reliability estimators in unidimensional continuous data. *Behavior Research Methods*, 56(6), 6299-6317.
- Cho, E. (2024b). The accuracy of reliability coefficients: A reanalysis of existing simulations. *Psychological Methods*, 29(2), 331-349.
- Cho, E., & Kim, S. (2015). Cronbach's coefficient alpha: Well known but poorly understood. *Organizational Research Methods*, 18(2), 207-230.
- Cronbach, L. J. (1951). Coefficient alpha and the internal structure of tests. *Psychometrika*, 16(3), 297-334
- Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255-282.
- Hoyt, C. (1941). Test reliability estimated by analysis of variance. *Psychometrika*, 6(3), 153-160.
- Kuder, G. F., & Richardson, M. W. (1937). The theory of the estimation of test reliability. *Psychometrika*, 2(3), 151-160.
- Novick, M. R., & Lewis, C. (1967). Coefficient alpha and the reliability of composite measurements. *Psychometrika*, 32(1), 1-13.
- Ten Berge, J. M. F., & Zegers, F. E. (1978). A series of lower bounds to the reliability of a test. *Psychometrika*, 43(4), 575-579.

See Also

[mu0()] alpha equals mu0.

Examples

```
alpha(Graham1)
alpha(Graham1, standardized = TRUE)
```

bifactor

Obtain bifactor reliability estimates

Description

Obtain bifactor reliability estimates. It is a multidimensional CFA reliability coefficient derived from the bifactor model. Items should be grouped by each sub-dimension.

Usage

```
bifactor(x, until, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
until a vector of indices indicating the last item of each sub-dimension
standardized If TRUE, the calculation is based on the correlation matrix.

Value

a list of class 'reliacoeff' containing bifactor reliability estimates

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

Examples

```
bifactor(Cho_multi, c(3, 6, 9))
```

 Cho2016

Cho's (2016) artificial unidimensional data of four items

Description

A hypothetical data consisting of four items.

A hypothetical data consisting of four items.

Usage

```
data(Cho2016)
```

```
data(Cho2016)
```

Format

An object of class `matrix` (inherits from `array`) with 4 rows and 4 columns.

An object of class `matrix` (inherits from `array`) with 4 rows and 4 columns.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

Source

Cho, E. (2016). Making reliability reliable: A systematic approach to reliability coefficients. *Organizational Research Methods*, 19(4), 651-682.

Cho, E. (2016). Making reliability reliable: A systematic approach to reliability coefficients. *Organizational Research Methods*, 19(4), 651-682.

 Cho_multi

Synthetic Multi-dimensional Covariance Matrix from Cho (2025)

Description

A 12x12 synthetic covariance matrix generated based on the simulation conditions described in Cho (2025). The data was generated as a random matrix with the following specifications: sample size (n) = 50, 4 group factors, and 3 items per group factor. The factor loading sizes within each factor are in a decreasing pattern. The general factor loading size relative to the group factor loading size is set to 'group', and there are no cross-loadings.

Usage

```
Cho_multi
```

Format

A 12x12 numeric matrix.

X11, X12, X13 Items for Factor 1

X21, X22, X23 Items for Factor 2

X31, X32, X33 Items for Factor 3

X41, X42, X43 Items for Factor 4

Source

Cho, E. (2025). Reliability and omega hierarchical in multidimensional data: A comparison of various estimators. *Psychological Methods*, 30(1), 40–59.

Examples

```
data(Cho_multi)
multirel(Cho_multi, until = c(3, 6, 9))
```

correlated_factors *Obtain correlated factors reliability estimates*

Description

Obtain correlated factors reliability estimates. It is a multidimensional CFA reliability coefficient derived from the correlated factors model. Items should be grouped by each sub-dimension.

Usage

```
correlated_factors(x, until, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
until a vector of indices indicating the last item of each sub-dimension
standardized If TRUE, the calculation is based on the correlation matrix.

Value

a list of class 'reliacoeff' containing correlated factors reliability estimates

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

Examples

```
correlated_factors(Cho_multi, c(3, 6, 9))
```

`feldt`*Obtain Feldt's classical congeneric reliability coefficient*

Description

Feldt's classical congeneric reliability (Feldt & Brennan 1989) is a unidimensional reliability coefficient based on a congeneric model. The congeneric model is a model that allows the length, discrimination, or importance of items to be different, and is the least restrictive model among the models derived from the classical test theory. The congeneric reliability proposed by Joreskog (1971) uses an optimization technique called maximum likelihood to estimate the "length" of an item. Classical congeneric reliability uses a simpler logic, using the ratio of the sum of the covariance of the item to the sum of the total covariance as an estimate of the length of the item (Cho 2016). This coefficient is slightly less accurate than the Gilmer-Feldt coefficient or congeneric reliability (Cho in press).

Usage

```
feldt(x, standardized = FALSE)
```

Arguments

`x` a dataframe or a matrix (unidimensional)
`standardized` If TRUE, the calculation is based on the correlation matrix.

Value

classical congeneric reliability coefficient

References

- Cho, E. (2016). Making reliability reliable: A systematic approach to reliability coefficients. *Organizational Research Methods*, 19(4), 651–682.
- Cho, E. (in press). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*.
- Feldt, L. S., & Brennan, R. L. (1989). Reliability. In R. L. Linn (Ed.), *Educational measurement* (3rd ed., pp. 105–146). American Council on Education and Macmillan.
- Jöreskog, K. G. (1971). Statistical analysis of sets of congeneric tests. *Psychometrika*, 36(2), 109–133.

See Also

[`gilmer()`] for the Gilmer-Feldt coefficient
[`joreskog()`] for congeneric (unidimensional CFA) reliability

Examples

```
feldt(Graham1)
```

`get_cov` *Obtain the Covariance Matrix*

Description

If the input data is a square and symmetric matrix, it is treated as an existing covariance/correlation matrix. Otherwise, a covariance matrix is calculated from the raw data.

Usage

```
get_cov(x, cor = FALSE)
```

Arguments

`x` A dataframe or a matrix.
`cor` If TRUE, return the correlation matrix. If FALSE, return the covariance matrix.

Value

The covariance or correlation matrix.

`gilmer` *Obtain the Gilmer-Feldt reliability coefficient*

Description

It is a unidimensional reliability coefficient based on a congeneric model. The congeneric model is a model that allows the length, discrimination, or importance of items to be different, and is the least restrictive model among the models derived from the classical test theory. The Gilmer-Feldt coefficient has the advantage of being less computational than congeneric reliability (Joreskog 1971) which uses confirmatory factor analysis. However, the Gilmer-Feldt coefficient derives a value very close to congeneric reliability (Cho in press). Feldt and Charter (2003) offers a user-friendly review of the Gilmer-Feldt coefficient.

Usage

```
gilmer(x, standardized = FALSE)
```

Arguments

`x` a data frame of raw data or a covariance matrix
`standardized` If TRUE, the calculation is based on the correlation matrix.

Value

The Gilmer-Feldt coefficient

References

Cho, E. (in press). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*.

Feldt, L. S., & Charter, R. A. (2003). Estimation of internal consistency reliability when test parts vary in effective length. *Measurement and Evaluation in Counseling and Development*, 36(1), 23-27

Gilmer, J. S., & Feldt, L. S. (1983). Reliability estimation for a test with parts of unknown lengths. *Psychometrika*, 48(1), 99–111.

Jöreskog, K. G. (1971). Statistical analysis of sets of congeneric tests. *Psychometrika*, 36(2), 109–133.

Examples

```
gilmer(Graham1)
```

Graham1

Graham's (2006) first unidimensional data

Description

Graham's (2006) artificial dataset consisting of five items. This is the second dataset in his Table 2. The last item is about five times as important as the other items. This dataset provides a good indication of how sensitive each reliability coefficient is to the violation of the tau equivalence assumption. The CFA reliability coefficient presented by the author is .97.

Graham's (2006) artificial dataset consisting of five items. This is the second dataset in his Table 2. The last item is about five times as important as the other items. This dataset provides a good indication of how sensitive each reliability coefficient is to the violation of the tau equivalence assumption. The CFA reliability coefficient presented by the author is .97.

Usage

```
data(Graham1)
```

```
data(Graham1)
```

Format

An object of class `matrix` (inherits from `array`) with 5 rows and 5 columns.

An object of class `matrix` (inherits from `array`) with 5 rows and 5 columns.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

Source

Graham, J. M. (2006). Congeneric and (essentially) tau-equivalent estimates of score reliability what they are and how to use them. *Educational and Psychological Measurement*, 66(6), 930-944. Education and Macmillan.

Graham, J. M. (2006). Congeneric and (essentially) tau-equivalent estimates of score reliability what they are and how to use them. *Educational and Psychological Measurement*, 66(6), 930-944. Education and Macmillan.

 hancock

Obtain Hancock's H (CFA version of maximal reliability)

Description

It is the confirmatory factor analysis (CFA) version of maximal reliability. This coefficient takes the standardized factor loading as the reliability of each item, and finds the weight that maximizes the reliability. Hence, Hancock's H shows a different result than the reliability estimator using conventional unit weights.

Usage

```
hancock(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

Hancock's H

References

Cho, E. (in press). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*.

Hancock, G., & Mueller, R. O. (2001). Rethinking construct reliability within latent variable systems. In R. Cudeck, S. du Toit, & D. Sörbom (Eds.), *Structural equation modeling: Present and future-A festschrift in honor of Karl Jöreskog* (pp. 195-216). Scientific Software International.

Li, H., Rosenthal, R., & Rubin, D. B. (1996). Reliability of measurement in psychology: From Spearman-Brown to maximal reliability. *Psychological Methods*, 1(1), 98-107.

McNeish, D. (2017). Thanks coefficient alpha, we'll take it from here. *Psychological Methods*, 23(3), 412-433.

Examples

```
hancock(Graham1)
```

`hancockan`*Obtain Hancock and An's (2020) coefficient*

Description

Hancock and An (2020) published a reliability estimator they called the "closed-form omega", which is designed to approximate a unidimensional confirmatory factor analysis reliability estimator (i.e., joreskog).

Usage

```
hancockan(x, standardized = FALSE)
```

Arguments

`x` a data frame of raw data or a covariance matrix
`standardized` If TRUE, the calculation is based on the correlation matrix.

Value

Hancock and An's reliability coefficient

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Hancock, G. R., & An, J. (2020). A Closed-Form Alternative for Estimating omega Reliability under Unidimensionality. *Measurement: Interdisciplinary Research and Perspectives*, 18(1), 1-14. <https://doi.org/10.1080/15366367.2019.1656049>

Examples

```
hancockan(Graham1)
```

heise	<i>Obtain Heise-Bohrnstedt's Omega</i>
-------	--

Description

Heise-Bohrnstedt's (1970) Omega is an factor analysis (FA) reliability. This formula is different from the FA reliability we use today and yields a larger value (Cho, in press). McDonald (1999) referred to all FA reliability as omega, and capitalized omega was used to distinguish it from McDonald's omega.

Usage

```
heise(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

Heise-Bohrnstedt's Omega

References

Cho, E. (in press). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*.

Heise, D. R., & Bohrnstedt, G. W. (1970). Validity, invalidity, and reliability. *Sociological Methodology*, 2, 104-129.

McDonald, R. P. (1999). *Test theory: A unified treatment*. Lawrence Erlbaum.

Examples

```
heise(Graham1)
```

joreskog	<i>Obtain Joreskog's congeneric reliability (Unidimensional CFA reliability)</i>
----------	--

Description

Congeneric reliability is a reliability coefficient derived from unidimensional confirmatory factor analysis (CFA).

Usage

```
joreskog(x, standardized = FALSE)
```

Arguments

`x` a data frame of raw data or a covariance matrix
`standardized` If TRUE, the calculation is based on the correlation matrix.

Details

Features: Congeneric reliability is a unidimensional reliability coefficient based on a unidimensional confirmatory factor analysis (CFA) model.

Name: Congeneric reliability is called by a variety of names, general users usually call it composite reliability, and reliability researchers often call it omega. One of the reasons for this confusion is that studies that first proposed this coefficient (Joreskog 1971) did not give this formula a name (Cho 2016). Joreskog (1971) proposed a matrix-form formula, and the commonly known non-matrix formula appears in Werts et al. (1974).

Frequency of use: Congeneric reliability is the second most commonly used reliability coefficient after coefficient alpha (Cho 2016)

Accuracy: Congeneric reliability is the most accurate reliability coefficient along with the Feldt-Gilmer coefficient (Cho in press)

Computation: This function uses maximum likelihood as estimation, unstandardized covariance matrix as input, and lavaan package as software.

Value

congeneric reliability coefficient

References

Cho, E. (2016). Making reliability reliable: A systematic approach to reliability coefficients. *Organizational Research Methods*, 19(4), 651-682.

Cho, E. (in press). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*.

Joreskog, K. G. (1971). Statistical analysis of sets of congeneric tests. *Psychometrika*, 36(2), 109-133.

Werts, C. E., Linn, R. L., & Joreskog, K. G. (1974). Intraclass reliability estimates: Testing structural assumptions. *Educational and Psychological Measurement*, 34, 25-33.

See Also

[`gilmer()`] for the Gilmer-Feldt coefficient
[`feldt()`] for classical congeneric reliability coefficient

Examples

```
joreskog(Graham1)
```

kaisercaffrey	<i>Obtain Kaiser-Caffrey's alpha (principal component analysis reliability)</i>
---------------	---

Description

Kaiser-Caffrey's (1965) alpha is the principal component analysis (PCA) reliability. They presented this formula in the context of factor analysis, but Bentler (1968) showed that it was in fact PCA reliability. Armor (1974), citing Bentler (1968), referred to this formula as theta, and some studies refer to it as Armor's theta. Kaiser and Caffrey (1965) labeled this formula alpha, and people may have mistaken it for coefficient alpha. See Vehkalahti (2000) and Cho(in press) for further explanation of this formula.

Usage

```
kaisercaffrey(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

Kaiser-Caffrey's alpha

References

- Armor, D. J. (1974). Theta reliability and factor scaling. In H. L. Costner (Ed.), *Sociological methodology* (pp. 17-50). Jossey-Bass.
- Bentler, P. M. (1968). Alpha-maximized factor analysis (alphamax) : Its relation to alpha and canonical factor analysis. *Psychometrika*, 33(3), 335-345.
- Cho, E. (in press). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*.
- Kaiser, H. F., & Caffrey, J. (1965). Alpha factor analysis. *Psychometrika*, 30(1), 1-14.
- Vehkalahti, K. (2000). Reliability of measurement scales: Tarkkonen's general method supersedes Cronbach's alpha. University of Helsinki.

Examples

```
kaisercaffrey(Graham1)
```

lambda2	<i>Obtain Ten Berge and Zegers' (1978) mu1</i>
---------	--

Description

Obtain Ten Berge and Zegers' (1978) mu1. mu1 equals Guttman's (1945) lambda2.

Usage

```
lambda2(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
standardized If TRUE, the calculation is based on the correlation matrix.

Value

Guttman's lambda2 (mu1) reliability estimate.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255-282.
Ten Berge, J. M. F., & Zegers, F. E. (1978). A series of lower bounds to the reliability of a test. *Psychometrika*, 43(4), 575-579.

lambda4_max	<i>Obtain the Maximum Split-Half Reliability (Lambda4)</i>
-------------	--

Description

Lambda4 is a lower bound of reliability based on a split-half of a test. This function searches through all possible split-halves to find the maximum value, which provides the best lower bound among all possible splits.

Usage

```
lambda4_max(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

The maximum lambda4 reliability estimate.

References

Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255-282.

 lambda5

Obtain Guttman's Lambda5

Description

Lambda5 is one of the six lower bounds of reliability established by Guttman (1945). It is particularly useful when one item has large absolute covariances with other items compared to the covariances among those items.

Usage

```
lambda5(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

Guttman's lambda5 reliability estimate.

References

Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255-282.

lambda6	<i>Obtain Guttman's Lambda6</i>
---------	---------------------------------

Description

Lambda6 is a lower bound of reliability based on linear multiple correlation. It tends to be larger than other bounds when items have relatively low zero-order intercorrelations but high multiple correlations.

Usage

```
lambda6(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

Guttman's lambda6 reliability estimate.

References

Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255-282.

maximal_reliability	<i>Obtain maximal reliability estimates of two-dimensional data</i>
---------------------	---

Description

Obtain maximal reliability estimates of two-dimensional data. Items should be grouped by each sub-dimension.

Usage

```
maximal_reliability(x, until, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 until a vector of indices indicating the last item of each sub-dimension
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

a list of class 'reliacoeff' containing the maximal reliability estimate

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Li, H., Rosenthal, R., & Rubin, D. B. (1996). Reliability of measurement in psychology: From Spearman-Brown to maximal reliability. *Psychological Methods*, 1(1), 98-107. <https://doi.org/10.1037/1082-989X.1.1.98>

Examples

```
maximal_reliability(Cho_multi, c(3, 6, 9))
```

mu1

Obtain Ten Berge and Zegers' (1978) mu1

Description

Obtain Ten Berge and Zegers' (1978) mu1. mu1 equals Guttman's (1945) lambda2.

Usage

```
mu1(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

Guttman's lambda2 (mu1) reliability estimate.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255-282.
 Ten Berge, J. M. F., & Zegers, F. E. (1978). A series of lower bounds to the reliability of a test. *Psychometrika*, 43(4), 575-579.

mu2

Obtain Ten Berge and Zegers' (1978) mu2

Description

Obtain Ten Berge and Zegers' (1978) mu2.

Usage

```
mu2(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
standardized If TRUE, the calculation is based on the correlation matrix.

Value

Ten Berge and Zegers' mu2 reliability estimate.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Ten Berge, J. M. F., & Zegers, F. E. (1978). A series of lower bounds to the reliability of a test. *Psychometrika*, 43(4), 575-579.

Examples

```
mu2(Graham1)
```

mu3

Obtain Ten Berge and Socan's (2004) mu3

Description

Obtain Ten Berge and Socan's (2004) mu3.

Usage

```
mu3(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Details

The original formula and the formula of psych's tenberge() are different. There is a high possibility that the original formula is incorrect and psych's version is correct. According to Equation (4) of the original article, μ_4 should increase monotonically (e.g., $\mu_4 \geq \mu_3$), but if the original formula is followed, it may decrease in some cases. The formula of the original paper is $2h$, but changing it to 2^h solves this problem. This function follows the latter interpretation.

Value

Ten Berge and Zegers' μ_3 reliability estimate.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Ten Berge, J. M. F., & Zegers, F. E. (1978). A series of lower bounds to the reliability of a test. *Psychometrika*, 43(4), 575-579.

Examples

```
mu3(Graham1)
```

mu4

Obtain Ten Berge and Socan's (2004) mu4

Description

Obtain Ten Berge and Socan's (2004) μ_4 .

Usage

```
mu4(x, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 standardized If TRUE, the calculation is based on the correlation matrix.

Details

The original formula and the formula of psych's `tenberge()` are different. There is a high possibility that the original formula is incorrect and psych's version is correct. According to Equation (4) of the original article, μ_4 should increase monotonically (e.g., $\mu_4 \geq \mu_3$), but if the original formula is followed, it may decrease in some cases. The formula of the original paper is $2h$, but changing it to 2^h solves this problem. This function follows the latter interpretation.

Value

Ten Berge and Zegers' μ_4 reliability estimate.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Ten Berge, J. M. F., & Zegers, F. E. (1978). A series of lower bounds to the reliability of a test. *Psychometrika*, 43(4), 575-579.

Examples

```
mu4(Graham1)
```

multirel

Compare multiple multidimensional reliability estimates

Description

This function calculates various multidimensional reliability coefficients including maximal reliability, correlated factors, stratified alpha, second-order factor, bifactor, and Nunnally's bottom-up approach.

Usage

```
multirel(x, until, standardized = FALSE, ...)
```

Arguments

<code>x</code>	a data frame of raw data or a covariance matrix
<code>until</code>	a vector of indices indicating the last item of each sub-dimension
<code>standardized</code>	If TRUE, the calculation is based on the correlation matrix.
<code>...</code>	Additional arguments passed to the underlying estimation functions.

Value

a list of class 'reliacoeff' containing multiple multidimensional estimates

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

Examples

```
multirel(Cho_multi, c(3, 6, 9))
```

multi_parallel

Obtain multidimensional parallel reliability

Description

Multidimensional parallel reliability is derived from the multidimensional parallel model (Cho, 2016). This is equivalent to entering the correlation instead of the covariance into the stratified alpha formula.

Usage

```
multi_parallel(x, until)
```

Arguments

x observed item scores or their covariances
until The number of items up to the first sub-construct

Value

a multidimensional parallel reliability estimate

References

Cho, E. (2016). Making reliability reliable: A systematic approach to reliability coefficients. *Organizational Research Methods*, 19(4), 651–682.

`nunnally`*Obtain bottom-up approach multidimensional reliability estimates*

Description

Among several approaches to estimating multidimensional reliability, these estimators use a bottom-up approach. That is, the test score is divided into sub-dimensional or sub-test scores. Multidimensional reliability is obtained by estimating the reliability of each subtest score and combining them. Different estimates can be obtained depending on how each subtest reliability is estimated. These estimators use the general formula first proposed by Jum Nunnally.

Usage

```
nunnally(x, until, method = "joreskog", standardized = FALSE)
```

Arguments

<code>x</code>	a data frame of raw data or a covariance matrix
<code>until</code>	a vector of indices indicating the last item of each sub-dimension
<code>method</code>	There are three options: "joreskog" (default), "mu" (uses mu4), and "kaisercaffrey".
<code>standardized</code>	If TRUE, the calculation is based on the correlation matrix.

Value

a list of class 'reliacoeff' containing bottom-up multidimensional reliability estimates

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Nunnally, J. C., & Bernstein, I. H. (1994). Psychometric theory (3rd ed). McGraw-Hill.

Examples

```
nunnally(Cho_multi, c(3, 6, 9), method = "mu")
nunnally(Cho_multi, c(3, 6, 9), method = "kaisercaffrey")
```

second_order *Obtain second-order factor reliability estimates*

Description

Obtain second-order factor reliability estimates. It is a multidimensional CFA reliability coefficient derived from the second-order factor model. Items should be grouped by each sub-dimension.

Usage

```
second_order(x, until, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 until a vector of indices indicating the last item of each sub-dimension
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

a list of class 'reliacof' containing second-order factor reliability estimates

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Cho, E. (2016). Making reliability reliable: A systematic approach to reliability coefficients. *Organizational Research Methods*, 19(4), 651-682. <https://doi.org/10.1177/1094428116656239>

Examples

```
second_order(Cho_multi, c(3, 6, 9))
```

simsek *Obtain Simsek-Noyan's theta (multidimensional PCA reliability)*

Description

Simsek-Noyan's (2013) theta is the multidimensional principal component analysis (PCA) reliability. It is a multidimensional generalization of Kaiser-Caffrey's alpha.

Usage

```
simsek(x, dim, standardized = FALSE)
```

Arguments

x a data frame of raw data or a covariance matrix
 dim the number of dimensions
 standardized If TRUE, the calculation is based on the correlation matrix.

Value

Simsek-Noyan's theta

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Armor, D. J. (1974). Theta reliability and factor scaling. In H. L. Costner (Ed.), *Sociological methodology* (pp. 17-50). Jossey-Bass.
 Kaiser, H. F., & Caffrey, J. (1965). Alpha factor analysis. *Psychometrika*, 30(1), 1-14.
 Simsek, G. G., & Noyan, F. (2013). McDonald's omega_t, Cronbach's alpha, and Generalized theta for Composite Reliability of Common Factors Structures. *Communications in Statistics - Simulation and Computation*, 42(9), 2008-2025. <https://doi.org/10.1080/03610918.2012.689062>

Examples

```
simsek(Cho_multi, dim = 4)
```

std_alpha	<i>Obtain standardized alpha</i>
-----------	----------------------------------

Description

Obtain standardized alpha

Usage

```
std_alpha(x)
```

Arguments

x a data frame of raw data or a covariance matrix

Value

standardized alpha reliability estimate

Examples

```
std_alpha(Graham1)
```

stratified_alpha	<i>Obtain stratified alpha reliability estimates</i>
------------------	--

Description

Obtain stratified alpha reliability estimates. It is a multidimensional version of coefficient alpha. Items should be grouped by each sub-dimension.

Usage

```
stratified_alpha(x, until, standardized = FALSE)
```

Arguments

x	a data frame of raw data or a covariance matrix
until	a vector of indices indicating the last item of each sub-dimension
standardized	If TRUE, the calculation is based on the correlation matrix.

Value

a list of class 'reliacoeff' containing stratified alpha reliability estimates

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Rajaratnam, N., Cronbach, L. J., & Gleser, G. C. (1965). Generalizability of stratified-parallel tests. *Psychometrika*, 30(1), 39–56. <https://doi.org/10.1007/BF02289746>

Cho, E. (2016). Making reliability reliable: A systematic approach to reliability coefficients. *Organizational Research Methods*, 19(4), 651-682. <https://doi.org/10.1177/1094428116656239>

Examples

```
stratified_alpha(Cho_multi, c(3, 6, 9))
```

test.tauequivalence *Test the essential tau-equivalence of the data*

Description

Compares goodness-of-fit indices between the essential tau-equivalence model and the congeneric model. It is used to test the assumption required for coefficient alpha and to investigate unidimensionality.

Usage

```
test.tauequivalence(data, standardized = FALSE)
```

Arguments

data a dataframe or a matrix (unidimensional)
standardized If TRUE, the calculation is based on the correlation matrix.

Value

a list of class 'reliacoeff' containing fit indices and chi-square difference test

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

Graham, J. M. (2006). Congeneric and (essentially) tau-equivalent estimates of score reliability what they are and how to use them. *Educational and Psychological Measurement*, 66(6), 930-944.

Examples

```
test.tauequivalence(Graham1)
```

unirel *Obtain Various Unidimensional Reliability Coefficients*

Description

This function calculates multiple reliability coefficients simultaneously.

Usage

```
unirel(x, standardized = FALSE, psych.include = TRUE)
```

Arguments

x	a data frame of raw data or a covariance matrix
standardized	If TRUE, calculations are based on the correlation matrix.
psych.include	Whether to include reliability coefficients (GLB.algebraic, GLB.fa) provided by the package psych.

Value

A list of class 'reliacoeff' containing reliability estimates.

Author(s)

Eunseong Cho, <bene@kw.ac.kr>

References

- Cho, E. (2021). Neither Cronbach's alpha nor McDonald's omega: A comment on Sijtsma and Pfadt. *Psychometrika*, 86(4), 877-886.
- Guttman, L. (1945). A basis for analyzing test-retest reliability. *Psychometrika*, 10(4), 255-282.

See Also

[alpha()] for coefficient alpha.

Examples

```
unirel(Graham1)
```

uni_cfa

Unidimensional confirmatory factor analysis

Description

Unidimensional confirmatory factor analysis

Usage

```
uni_cfa(  
  sigma,  
  what = "est",  
  sample_size = 500,  
  nonneg_error = TRUE,  
  taueq = FALSE,  
  parallel = FALSE  
)
```

Arguments

<code>sigma</code>	a covariance matrix
<code>what</code>	e.g., "est", "std", "fit"
<code>sample_size</code>	number of sample observations
<code>nonneg_error</code>	if TRUE, constraint error variances to positive values
<code>taueq</code>	if TRUE, a tau-equivalent model is estimated
<code>parallel</code>	if TRUE, a parallel model is estimated

Value

parameter estimates of unidimensional cfa model

Examples

```
uni_cfa(Graham1)
```

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